BETH 2313 - Fluid Mechanics

Chapter 5 (Continued)
MASS, BERNOULLI AND ENERGY EQUATIONS

Lecture slides by

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5–3 ■ MECHANICAL ENERGY AND EFFICIENCY

Mechanical energy: The form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device such as an ideal turbine.

Mechanical energy of a flowing fluid per unit mass:

$$e_{\text{mech}} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

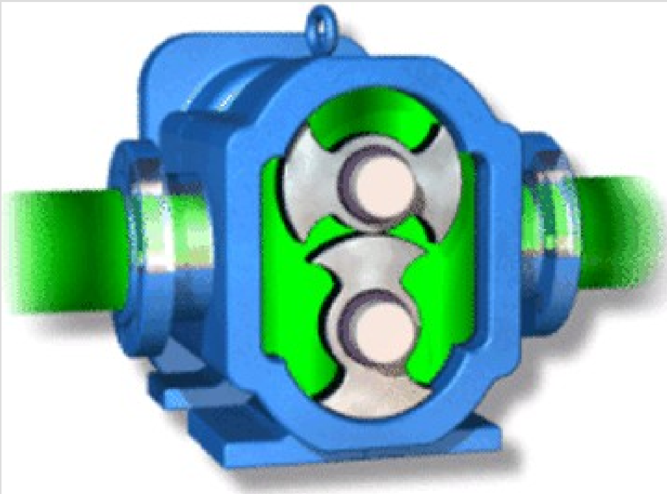
Flow energy + kinetic energy + potential energy

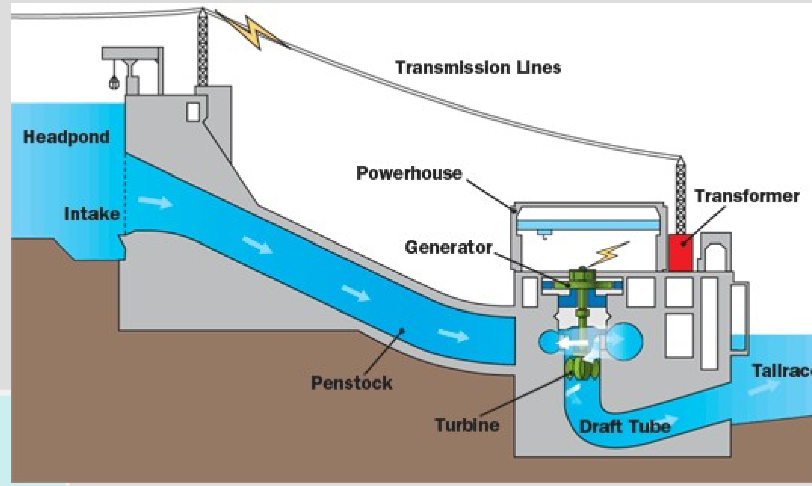
Mechanical energy change:

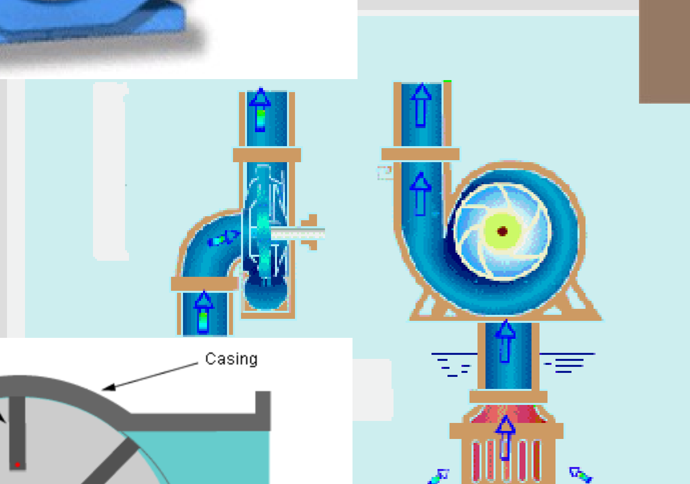
$$\Delta e_{\text{mech}} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad \text{(kJ/kg)}$$

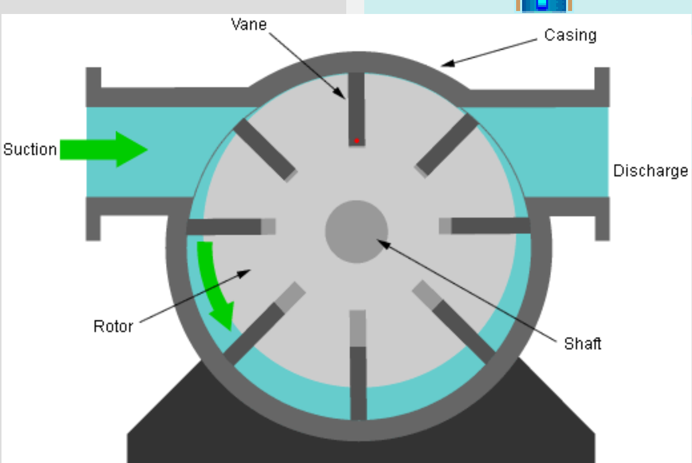
- The mechanical energy of a fluid does not change during flow if its pressure, density, velocity, and elevation remain constant.
- In the absence of any irreversible losses, the mechanical energy change represents the mechanical work supplied to the fluid (if $\Delta e_{\text{mech}} > 0$) or extracted from the fluid (if $\Delta e_{\text{mech}} < 0$).

Different Between Pump & Turbine









Different Between Pump & Turbine

Shaft work: The transfer of mechanical energy is usually accomplished by a rotating shaft, and thus mechanical work is often referred to as *shaft work*.

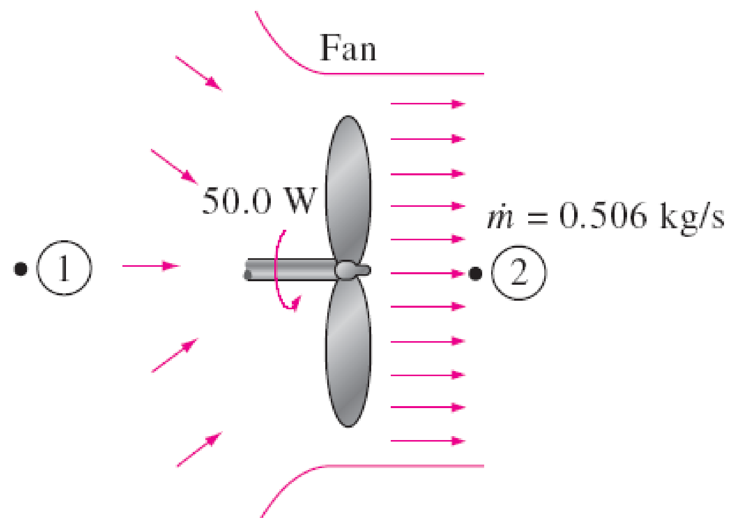
A pump or a fan receives shaft work (usually from an electric motor) and transfers it to the fluid as mechanical energy (less frictional losses).

A turbine converts the mechanical energy of a fluid to shaft work.

Mechanical efficiency of a device or process

The effectiveness of the conversion process between the mechanical work supplied or extracted and the mechanical energy of the fluid is expressed by the **pump efficiency** and **turbine efficiency**,

$$\eta_{mech} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy input}} = \frac{E_{mech,\,out}}{E_{mech,\,in}} = 1 - \frac{E_{mech,\,loss}}{E_{mech,\,in}}$$

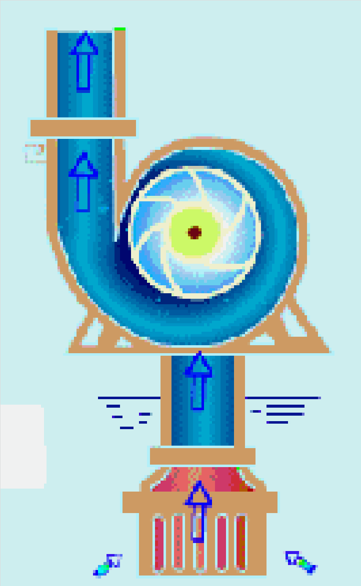


$V_1 \times 0,\ V_2 = 12.1$ m/s
$z_1 = z_2$
$P_1 \times P_{atm}$ and $P_2 \times P_{atm}$

$$\eta_{mech,\,fan} = \frac{\Delta \dot{E}_{mech,\,fluid}}{\dot{W}_{shaft,\,in}} = \frac{\dot{m}V_2^{\,2}/2}{\dot{W}_{shaft,\,in}} = \frac{(0.506 \text{ kg/s})(12.1 \text{ m/s})^2/2}{50.0 \text{ W}} = 0.741$$

The mechanical efficiency of a fan is the ratio of the rate of increase of the mechanical energy of the air to the mechanical power input.

A pump or a fan receives shaft work (usually from an electric motor) and transfers it to the fluid as mechanical energy (less frictional losses).



$$\eta_{\text{pump}} = \frac{\text{Mechanical power increase of the fluid}}{\text{Mechanical power input}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{shaft, in}}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{pump}}}$$

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}}$$

A turbine converts the mechanical energy of a fluid to shaft work.

$$\eta_{\text{turbine}} = \frac{\text{Mechanical power output}}{\text{Mechanical power decrease of the fluid}} = \frac{\dot{W}_{\text{shaft, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine, }e}}$$

$$|\Delta \dot{E}_{\text{mech, fluid}}| = \dot{E}_{\text{mech, in}} - \dot{E}_{\text{mech, out}}$$

$$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electric power input}} = \frac{\dot{W}_{\text{shaft,out}}}{\dot{W}_{\text{elect,in}}}$$

Motor efficiency

$$\eta_{\text{generator}} = \frac{\text{Electric power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{shaft,in}}}$$

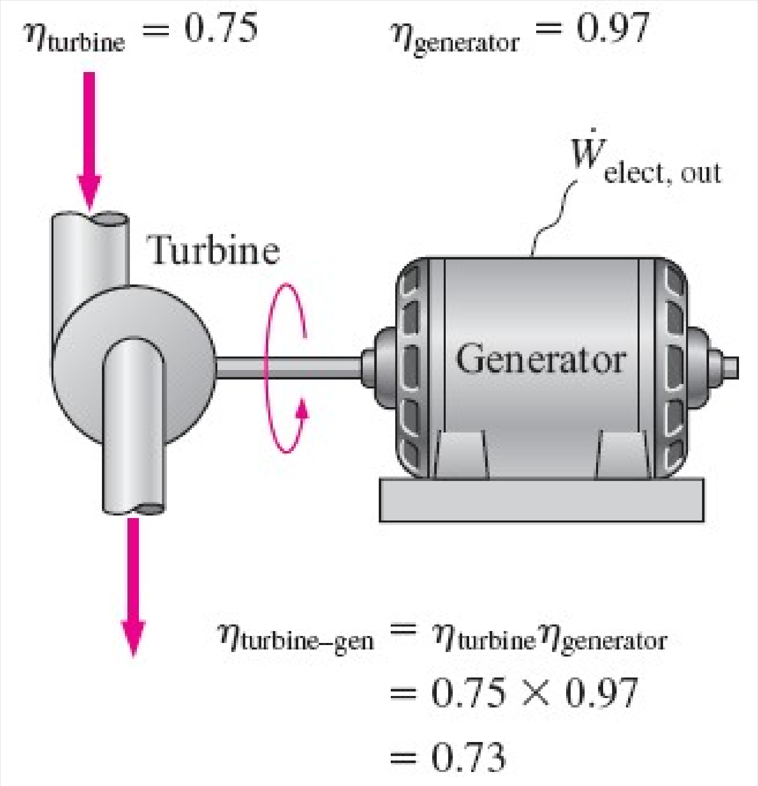
Generator efficiency

$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = \frac{\dot{W}_{\text{pump},u}}{\dot{W}_{\text{elect,in}}} = \frac{\Delta \dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}}$$

Pump-Motor overall efficiency

Turbine-Generator overall efficiency:

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\eta_{\text{generator}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{turbine},e}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta \dot{E}_{\text{mech,fluid}}|}$$



The overall efficiency of a turbine–generator is the product of the efficiency of the turbine and the efficiency of the generator, and represents the fraction of the mechanical energy of the fluid converted to electric energy.

The efficiencies just defined range between 0 and 100%.

0% corresponds to the conversion of the entire mechanical or electric energy input to thermal energy, and the device in this case functions like a resistance heater.

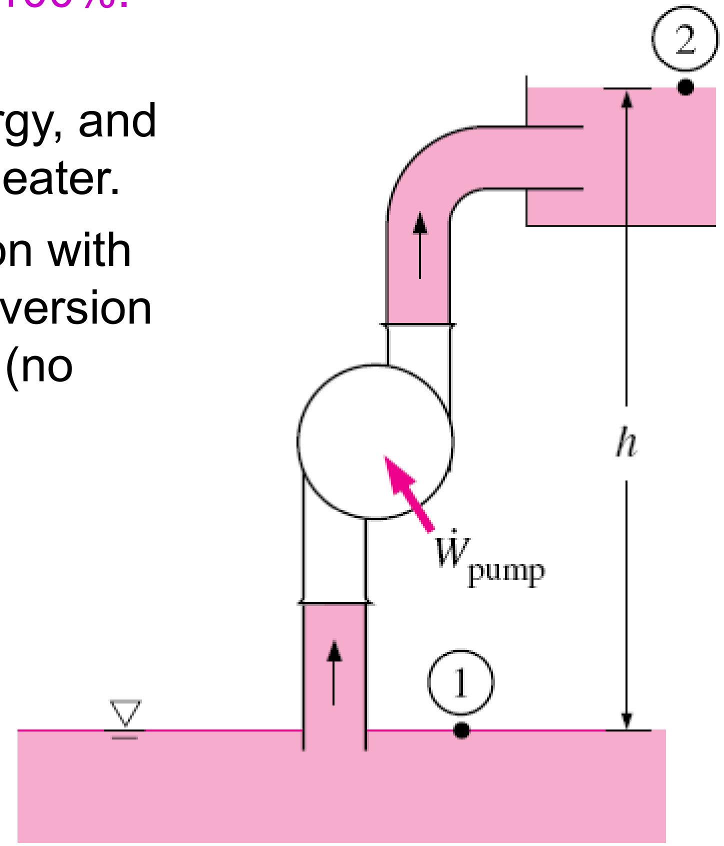
100% corresponds to the case of perfect conversion with no friction or other irreversibilities, and thus no conversion of mechanical or electric energy to thermal energy (no losses).

For systems that involve only *mechanical forms of energy* and its transfer as *shaft work*, the conservation of energy is

$$E_{\text{mech, in}} - E_{\text{mech, out}} = \Delta E_{\text{mech, system}} + E_{\text{mech, loss}}$$

***E*mech, loss** : The conversion of mechanical energy to thermal energy due to irreversibilities such as friction.

Many fluid flow problems involve mechanical forms of energy only, and such problems are conveniently solved by using a *mechanical energy* balance.



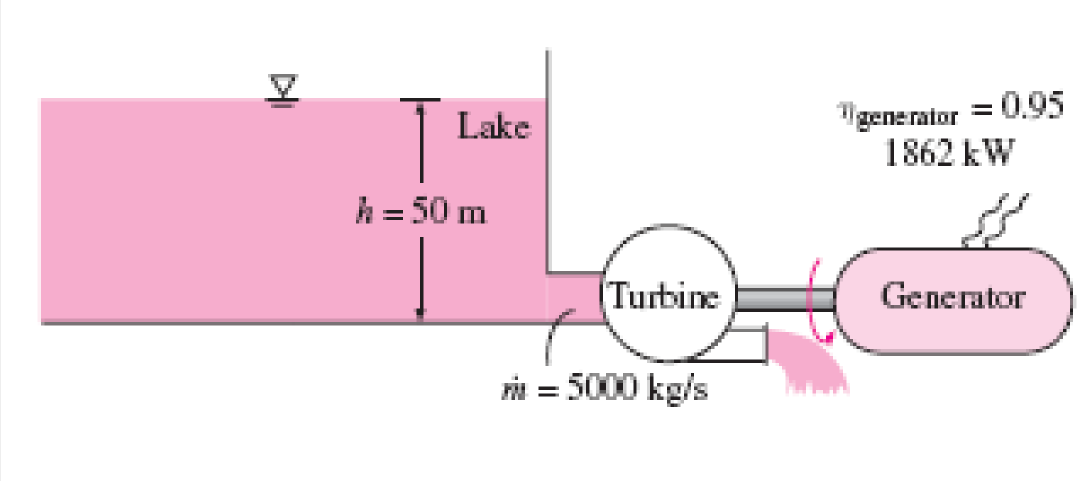
Steady flow

$$V_1 = V_2 \approx 0$$
$$z_2 = z_1 + h$$
$$P_1 = P_2 = P_{\text{atm}}$$

$$\dot{E}_{\text{mech, in}} = \dot{E}_{\text{mech, out}} + \dot{E}_{\text{mech, loss}}$$
$$\dot{W}_{\text{pump}} + \dot{m}gz_1 = \dot{m}gz_2 + \dot{E}_{\text{mech, loss}}$$
$$\dot{W}_{\text{pump}} = \dot{m}gh + \dot{E}_{\text{mech, loss}}$$

EXAMPLE 5–3 **Performance of a Hydraulic Turbine–Generator**

The water in a large lake is to be used to generate electricity by the installation of a hydraulic turbine–generator at a location where the depth of the water is 50 m (Fig. 5–18). Water is to be supplied at a rate of 5000 kg/s. If the electric power generated is measured to be 1862 kW and the generator efficiency is 95 percent, determine (*a*) the overall efficiency of the turbine–generator, (*b*) the mechanical efficiency of the turbine, and (*c*) the shaft power supplied by the turbine to the generator.



Assumptions **1** The elevation of the lake remains constant. **2** The mechanical energy of water at the turbine exit is negligible.

Properties The density of water can be taken to be $\rho = 1000 \text{ kg/m}^3$.

Analysis (*a*) We take the bottom of the lake as the reference level for convenience. Then kinetic and potential energies of water are zero, and the change in its mechanical energy per unit mass becomes

$$e_{\text{mech, in}} - e_{\text{mech, out}} = \frac{P}{\rho} - 0 = gh = (9.81 \text{ m/s}^2)(50 \text{ m})\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 0.491 \text{ kJ/kg}$$

Then the rate at which mechanical energy is supplied to the turbine by the fluid and the overall efficiency become

$$|\Delta \dot{E}_{\text{mech, fluid}}| = \dot{m}(e_{\text{mech, in}} - e_{\text{mech, out}}) = (5000 \text{ kg/s})(0.491 \text{ kJ/kg}) = 2455 \text{ kW}$$

$$\eta_{\text{overall}} = \eta_{\text{turbine–gen}} = \frac{\dot{W}_{\text{elect, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{1862 \text{ kW}}{2455 \text{ kW}} = \mathbf{0.76}$$

(*b*) Knowing the overall and generator efficiencies, the mechanical efficiency of the turbine is determined from

$$\eta_{\text{turbine–gen}} = \eta_{\text{turbine}}\,\eta_{\text{generator}} \rightarrow \eta_{\text{turbine}} = \frac{\eta_{\text{turbine–gen}}}{\eta_{\text{generator}}} = \frac{0.76}{0.95} = \mathbf{0.80}$$

(*c*) The shaft power output is determined from the definition of mechanical efficiency,

$$\dot{W}_{\text{shaft, out}} = \eta_{\text{turbine}}|\Delta \dot{E}_{\text{mech, fluid}}| = (0.80)(2455 \text{ kW}) = \mathbf{1964 \text{ kW}}$$

5–19 Consider a river flowing toward a lake at an average velocity of 3 m/s at a rate of 500 m_3/s at a location 90 m above the lake surface. Determine the total mechanical energy of the river water per unit mass and the power generation potential of the entire river at that location. (*Ans:* 444 MW)

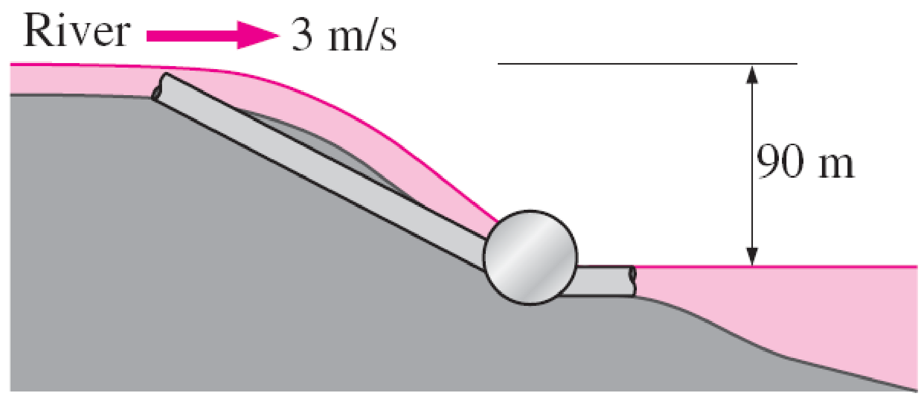


FIGURE P5–19

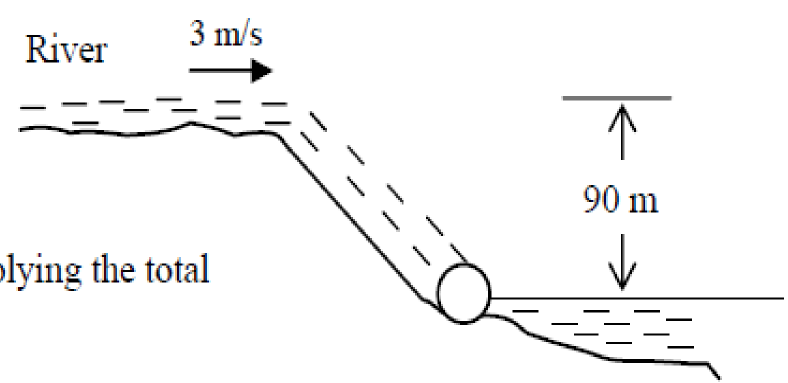
Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis Noting that the sum of the flow energy and the potential energy is constant for a given fluid body, we can take the elevation of the entire river water to be the elevation of the free surface, and ignore the flow energy. Then the total mechanical energy of the river water per unit mass becomes

$$e_{\text{mech}} = pe + ke = gh + \frac{V^2}{2}$$

$$= \left((9.81\ \text{m/s}^2)(90\ \text{m}) + \frac{(3\ \text{m/s})^2}{2} \right)\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2} \right)$$

$$= 0.887\ \text{kJ/kg}$$



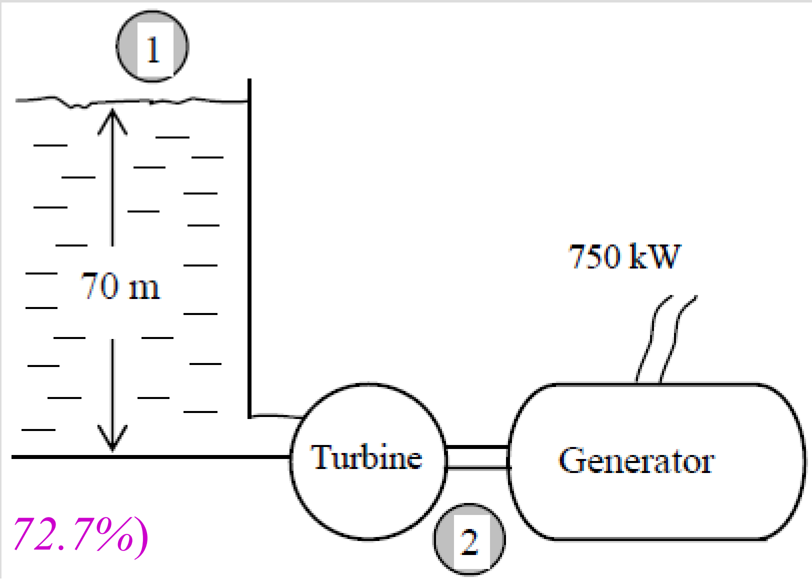
The power generation potential of the river water is obtained by multiplying the total mechanical energy by the mass flow rate,

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(500\ \text{m}^3/\text{s}) = 500{,}000\ \text{kg/s}$$

$$\dot{W}_{\text{max}} = \dot{E}_{\text{mech}} = \dot{m}e_{\text{mech}} = (500{,}000\ \text{kg/s})(0.887\ \text{kg/s}) = 444{,}000\ \text{kW} = \mathbf{444\ \ MW}$$

5–20 Electric power is to be generated by installing a hydraulic turbine–generator at a site 70 m below the free surface of a large water reservoir that can supply water at a rate of 1500 kg/s steadily. If the mechanical power output of the turbine is 800 kW and the electric power generation is 750 kW, determine the turbine efficiency and the combined turbine–generator efficiency of this plant. Neglect losses in the pipes.

(Ans: 77.6%, 72.7%)



Analysis We take the free surface of the reservoir to be point 1 and the turbine exit to be point 2. We also take the turbine exit as the reference level ($z_2 = 0$), and thus the potential energy at points 1 and 2 are $pe_1 = gz_1$ and $pe_2 = 0$. The flow energy P/ρ at both points is zero since both 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$). Further, the kinetic energy at both points is zero ($ke_1 = ke_2 = 0$) since the water at point 1 is essentially motionless, and the kinetic energy of water at turbine exit is assumed to be negligible. The potential energy of water at point 1 is

$$pe_1 = gz_1 = (9.81\,\text{m/s}^2)(70\,\text{m})\left(\frac{1\,\text{kJ/kg}}{1000\,\text{m}^2/\text{s}^2}\right) = 0.687\,\text{kJ/kg}$$

Then the rate at which the mechanical energy of the fluid is supplied to the turbine become

$$\begin{aligned}\left|\Delta\dot{E}_{\text{mech,fluid}}\right| &= \dot{m}(e_{\text{mech,in}} - e_{\text{mech,out}}) = \dot{m}(pe_1 - 0) = \dot{m}pe_1 \\ &= (1500\,\text{kg/s})(0.687\,\text{kJ/kg}) \\ &= 1031\,\text{kW}\end{aligned}$$

$$\eta_{\text{turbine-gen}} = \frac{W_{\text{elect,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|} = \frac{750\,\text{kW}}{1031\,\text{kW}} = 0.727 \quad \text{or} \quad \mathbf{72.7\%}$$

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|} = \frac{800\,\text{kW}}{1031\,\text{kW}} = 0.776 \quad \text{or} \quad \mathbf{77.6\%}$$

5–24 Water is pumped from a lake to a storage tank 20 m above at a rate of 0.07 m^3/s while consuming 20.4 kW of electric power. Disregarding any frictional losses in the pipes and any changes in kinetic energy, determine

a) the overall efficiency of the pump–motor unit.

b) the pressure difference between the inlet and the exit of the pump.

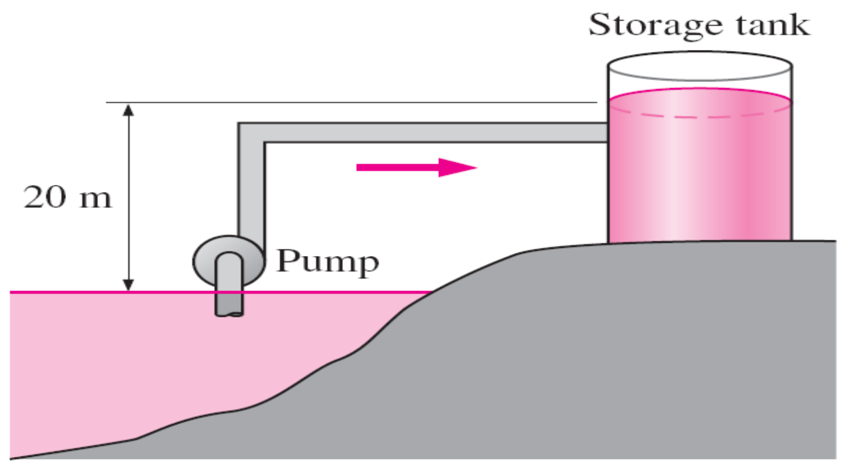


FIGURE P5–24

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(0.070\ \text{m}^3/\text{s}) = 70\ \text{kg/s}$$

$$pe_1 = gz_1 = (9.81\ \text{m/s}^2)(20\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.196\ \text{kJ/kg}$$

Then the rate of increase of the mechanical energy of water becomes

$$\Delta\dot{E}_{\text{mech,fluid}} = \dot{m}(e_{\text{mech,out}} - e_{\text{mech,in}}) = \dot{m}(pe_2 - 0) = \dot{m}pe_2 = (70\ \text{kg/s})(0.196\ \text{kJ/kg}) = 13.7\ \text{kW}$$

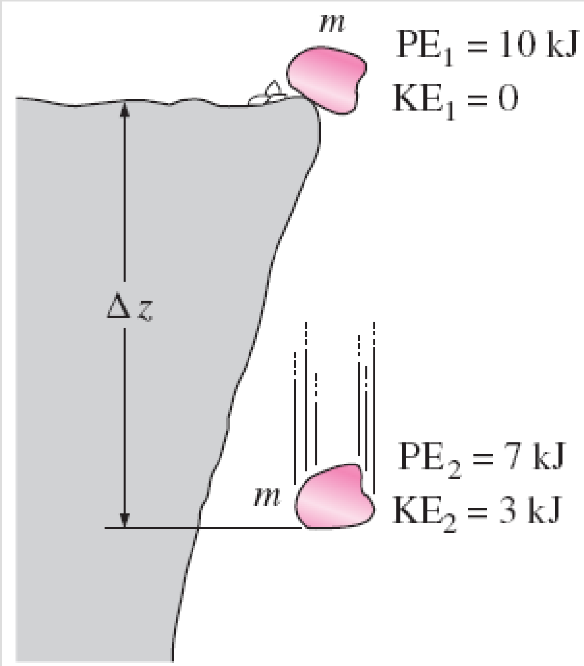
The overall efficiency of the combined pump-motor unit is determined from its definition,

$$\eta_{\text{pump-motor}} = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}} = \frac{13.7\ \text{kW}}{20.4\ \text{kW}} = 0.672 \quad \text{or} \quad \mathbf{67.2\%}$$

$$\Delta\dot{E}_{\text{mech,fluid}} = \dot{m}(e_{\text{mech,out}} - e_{\text{mech,in}}) = \dot{m}\frac{P_2 - P_1}{\rho} = \dot{V}\Delta P$$

$$\Delta P = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{V}} = \frac{13.7\ \text{kJ/s}}{0.070\ \text{m}^3/\text{s}}\left(\frac{1\ \text{kPa}\cdot\text{m}^3}{1\ \text{kJ}}\right) = \mathbf{196\ kPa}$$

5–5 ■ GENERAL ENERGY EQUATION



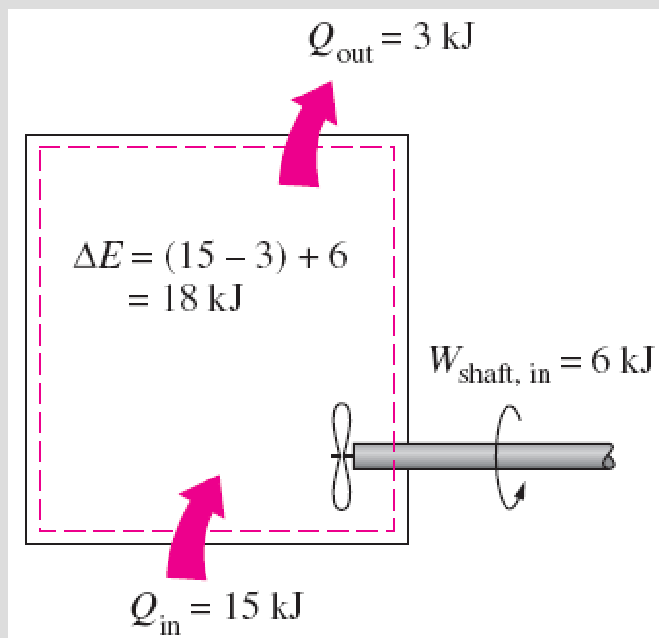
The first law of thermodynamics (the conservation of energy principle): Energy cannot be created or destroyed during a process; it can only change forms.

$$E_{in} - E_{out} = \Delta E$$

$$\dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{dE_{sys}}{dt} \qquad \dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{d}{dt}\int_{sys} \rho e\, dV$$

$$\dot{Q}_{net\ in} = \dot{Q}_{in} - \dot{Q}_{out} \qquad \dot{W}_{net\ in} = \dot{W}_{in} - \dot{W}_{out}$$

$$\boxed{e = u + \mathrm{ke} + \mathrm{pe} = u + \frac{V^2}{2} + gz}$$



The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.

Energy Transfer by Heat, *Q*

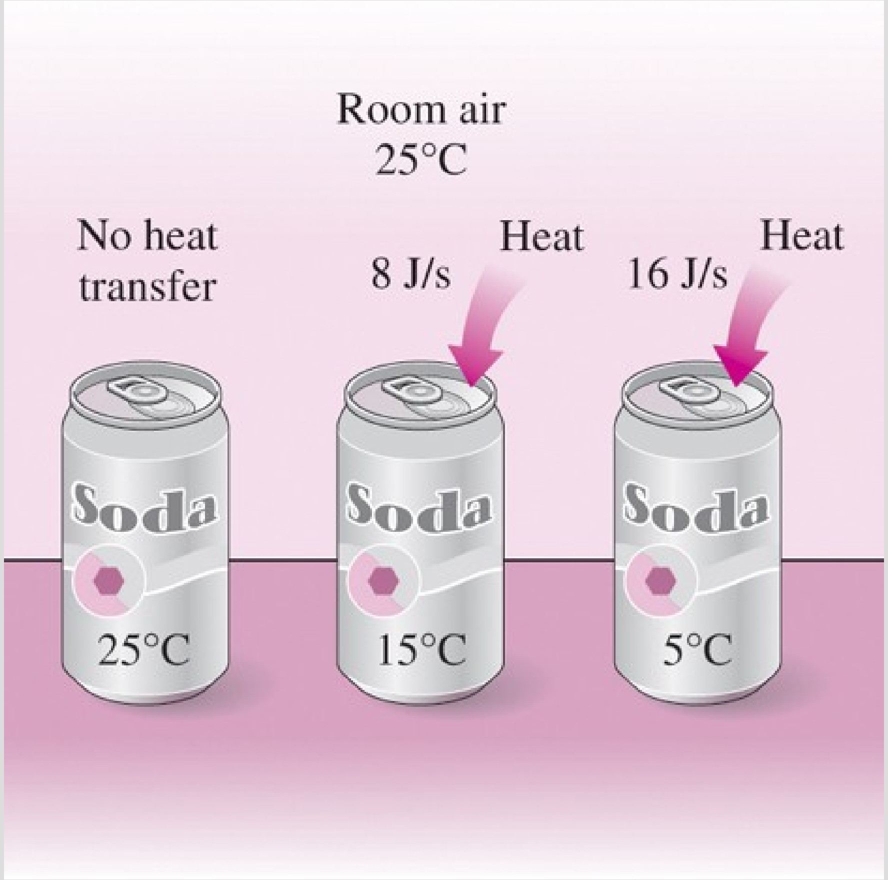
Thermal energy: The sensible and latent forms of internal energy.

Heat Transfer: The transfer of energy from one system to another as a result of a temperature difference.

The direction of heat transfer is always from the higher-temperature body to the lower-temperature one.

Adiabatic process: A process during which there is no heat transfer.

Heat transfer rate: The time rate of heat transfer.



Temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer.

Energy Transfer by Work, *W*

- **Work**: The energy transfer associated with a force acting through a distance.
- A rising piston, a rotating shaft, and an electric wire crossing the system boundaries are all associated with work interactions.
- **Power:** The time rate of doing work.
- Car engines and hydraulic, steam, and gas turbines produce work; compressors, pumps, fans, and mixers consume work.

$$W_{\text{total}} = W_{\text{shaft}} + W_{\text{pressure}} + W_{\text{viscous}} + W_{\text{other}}$$

W_{shaft} The work transmitted by a rotating shaft

$W_{pressure}$ The work done by the pressure forces on the control surface

$W_{viscous}$ The work done by the normal and shear components of viscous forces on the control surface

W_{other} The work done by other forces such as electric, magnetic, and surface tension

Shaft Work

A force F acting through a moment arm r generates a torque T

$$\mathrm{T} = Fr \quad \rightarrow \quad F = \frac{\mathrm{T}}{r}$$

This force acts through a linear distance s

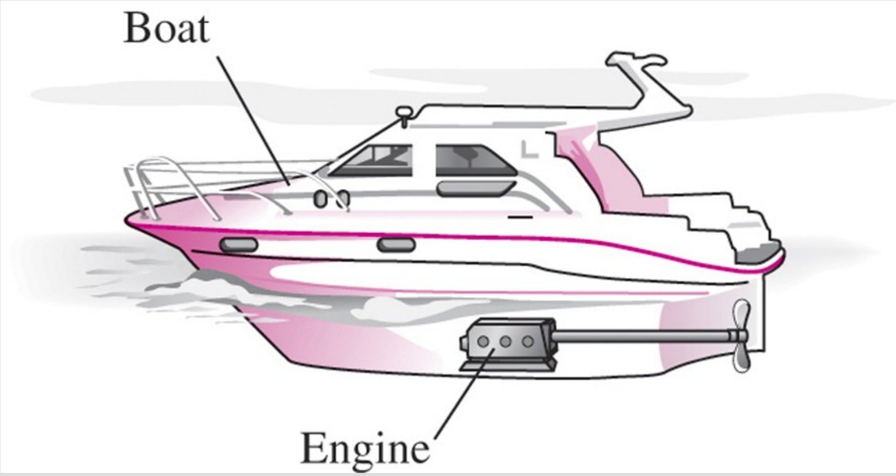
$$s = (2\pi r)n$$

Shaft work

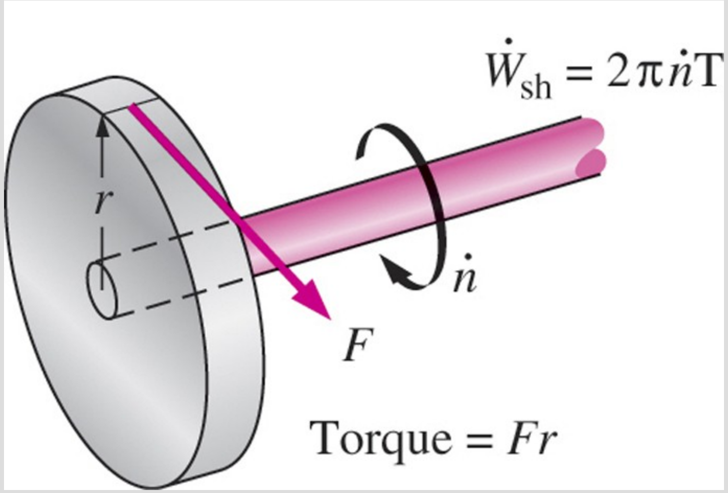
$$W_{sh} = Fs = \left(\frac{\mathrm{T}}{r}\right)(2\pi rn) = 2\pi n\mathrm{T} \qquad (\mathrm{kJ})$$

The power transmitted through the shaft is the shaft work done per unit time:

$$\boxed{\dot{W}_{shaft} = \omega \mathrm{T}_{shaft} = 2\pi \dot{n} \mathrm{T}_{shaft} \quad (\mathrm{kW})}$$



Energy transmission through rotating shafts is commonly encountered in practice.



Shaft work is proportional to the torque applied and the number of revolutions of the shaft.

Work Done by Pressure Forces

$$\delta W_{\text{boundary}} = PA\ ds$$

$$\delta \dot{W}_{\text{pressure}} = \delta \dot{W}_{\text{boundary}} = PAV_{\text{piston}} \qquad V_{\text{piston}} = ds/dt$$

$$\delta \dot{W}_{\text{pressure}} = -P\ dA\ V_n = -P\ dA(\vec{V} \cdot \vec{n})$$

$$\dot{W}_{\text{pressure, net in}} = -\int_A P(\vec{V} \cdot \vec{n})\ dA = -\int_A \frac{P}{\rho} \rho(\vec{V} \cdot \vec{n})\ dA$$

$$\dot{W}_{\text{net in}} = \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \dot{W}_{\text{shaft, net in}} - \int_A P(\vec{V} \cdot \vec{n})\ dA$$

Piston Power

$$F = PA$$

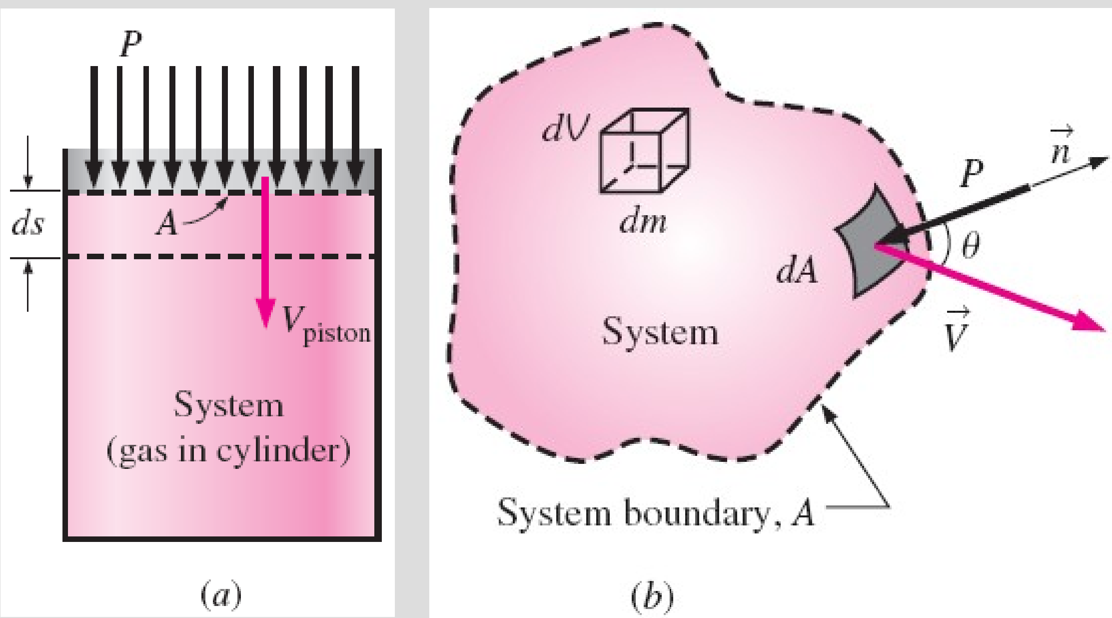
$$F = P(\pi r^2)$$

$$W = Fs = P(\pi r^2)h$$

$$\boxed{\dot{W}_P = (\pi r^2)P h \dot{n}}$$

or

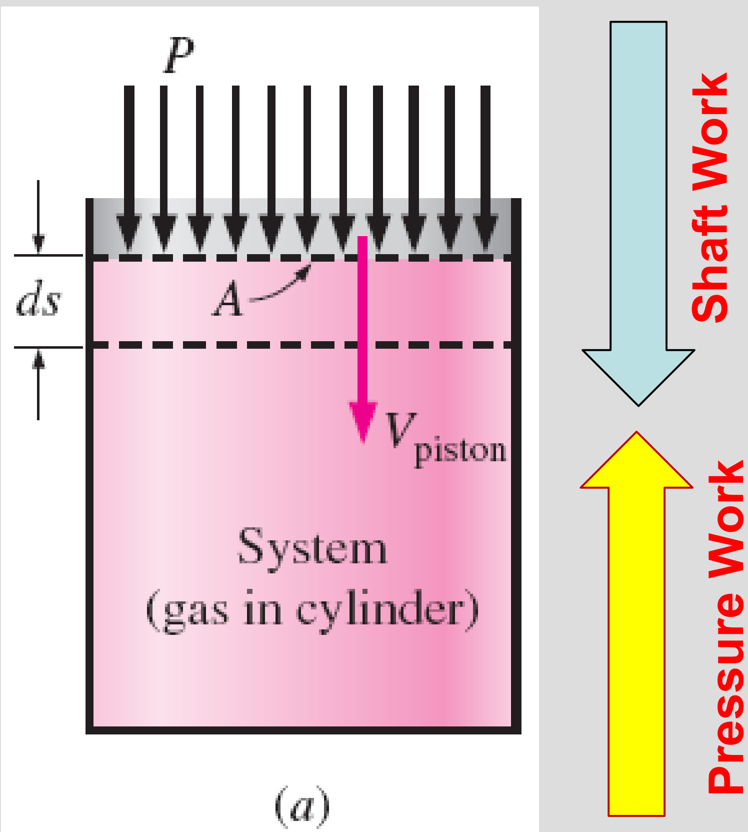
$$F = PAV$$



The pressure force acting on (*a*) the moving boundary of a system in a piston-cylinder device, and (*b*) the differential surface area of a system of arbitrary shape.

Work Done by Pressure Forces

$$\dot{W}_{\text{net in}} = \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \dot{W}_{\text{shaft, net in}} - \int_{A} P(\vec{V} \cdot \vec{n})\, dA$$



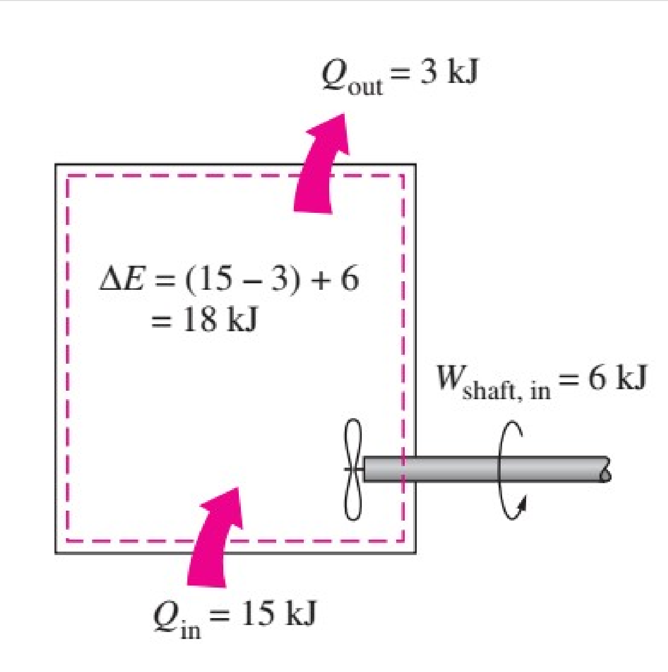
Why should -ve

General Energy Equation

Energy content of a fixed quantity of mass (a closed system) can be
changed by two mechanisms:

a) heat transfer Q

b) work transfer W.



The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.

Conservation of energy (fixed mass):

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{dE_{\text{sys}}}{dt}$$

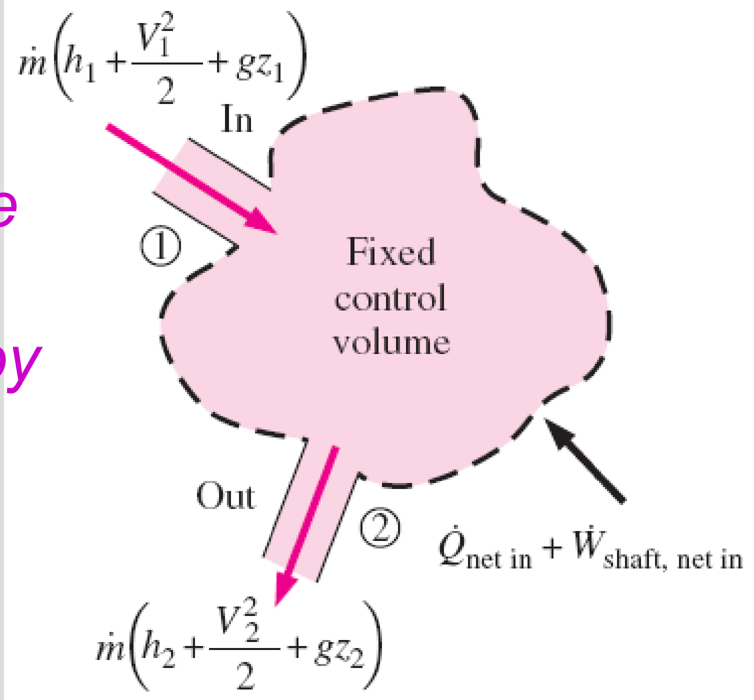
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{\text{sys}} \rho e \, dV$$

$$\underbrace{e}_{E_{\text{net}}} = \overbrace{u}^{E_{\text{internal}}} + \underbrace{\text{ke}}_{E_{\text{kinetic}}} + \overbrace{\text{pe}}^{E_{\text{potential}}} = u + \frac{V^2}{2} + gz$$

5–6 ■ ENERGY ANALYSIS OF STEADY FLOWS

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

The net rate of energy transfer to a control volume by heat transfer and work during steady flow is equal to the difference between the rates of outgoing and incoming energy flows by mass flow.



A control volume with only one inlet and one outlet and energy interactions.

single-stream devices (one inlet & one outlet), ie: constant mass

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

per unit mass

$$h = u + Pv = u + P/\rho$$

5–6 ■ ENERGY ANALYSIS OF STEADY FLOWS

single-stream devices (one inlet & one outlet), ie: constant mass

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$h = u + P/\rho$$

$$\underbrace{w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1}_{E_{\text{mech. input}}} = \underbrace{\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2}_{E_{\text{mech. output}}} + \underbrace{(u_2 - u_1 - q_{\text{net in}})}_{E_{\text{mech. loss}}}$$

$$w_{\text{shaft, net in}} = \underbrace{\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}}_{E_{\text{flow}}} + \underbrace{\frac{V_2^2 - V_1^2}{2}}_{E_{\text{kinetic}}} + \overbrace{g(z_2 - z_1)}^{E_{\text{potential}}} + \underbrace{(u_2 - u_1 - q_{\text{net in}})}_{E_{\text{internal}}}$$

Ideal flow (no mechanical energy loss):

$$q_{\text{net in}} = u_2 - u_1$$

Real flow (with mechanical energy loss):

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}}$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech, loss}}$$

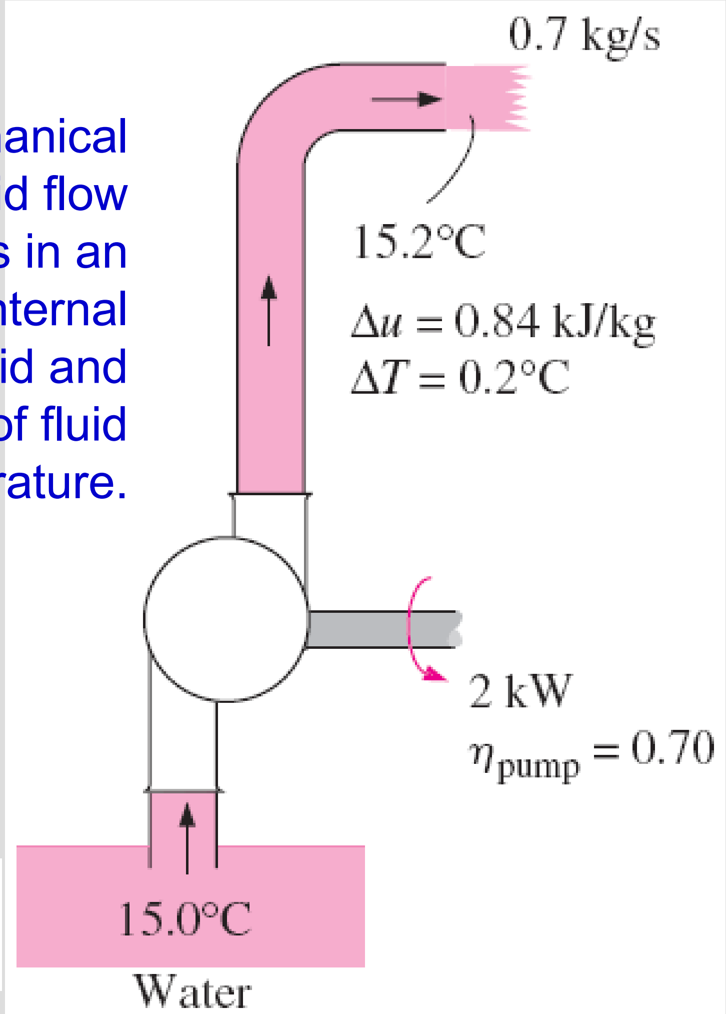
$$w_{\text{shaft, net in}} = w_{\text{pump}} - w_{\text{turbine}}$$

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

$$\boxed{\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}}$$

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$

The lost mechanical energy in a fluid flow system results in an increase in the internal energy of the fluid and thus in a rise of fluid temperature.



Energy equation in terms of *heads (previous eq. divide by g)*

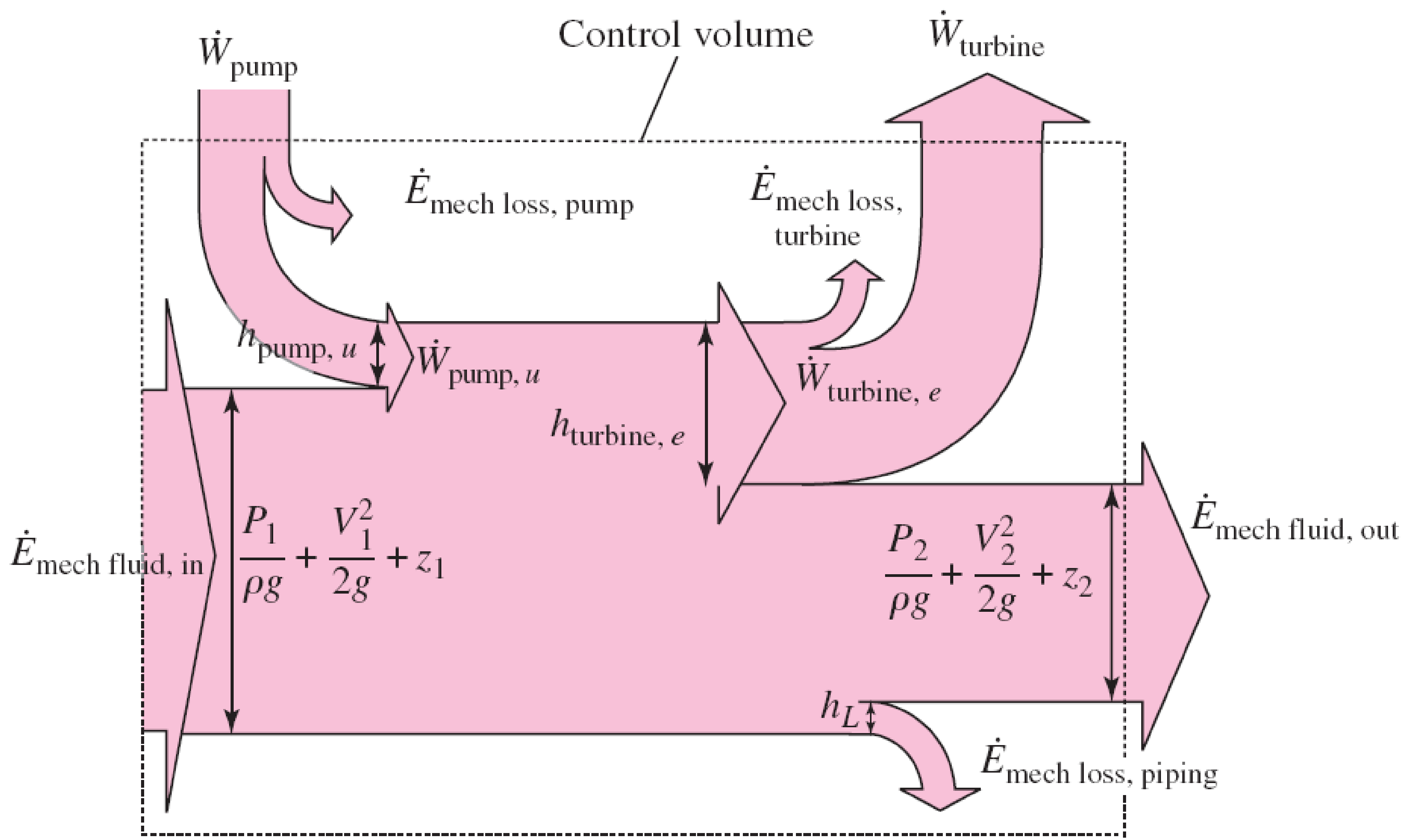
$$\boxed{\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L}$$

where

- $h_{\text{pump}, u} = \dfrac{w_{\text{pump}, u}}{g} = \dfrac{\dot{W}_{\text{pump}, u}}{\dot{m} g} = \dfrac{\eta_{\text{pump}} \dot{W}_{\text{pump}}}{\dot{m} g}$ is the *useful head delivered to the fluid by the pump*. Because of irreversible losses in the pump, $h_{\text{pump}, u}$ is *less* than $\dot{W}_{\text{pump}}/\dot{m}g$ by the factor η_{pump}.

- $h_{\text{turbine}, e} = \dfrac{w_{\text{turbine}, e}}{g} = \dfrac{\dot{W}_{\text{turbine}, e}}{\dot{m} g} = \dfrac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}} \dot{m} g}$ is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, $h_{\text{turbine}, e}$ is greater than $\dot{W}_{\text{turbine}}/\dot{m}g$ by the factor η_{turbine}.

- $h_L = \dfrac{e_{\text{mech loss, piping}}}{g} = \dfrac{\dot{E}_{\text{mech loss, piping}}}{\dot{m} g}$ is the irreversible *head loss* between 1 and 2 due to all components of the piping system other than the pump or turbine.

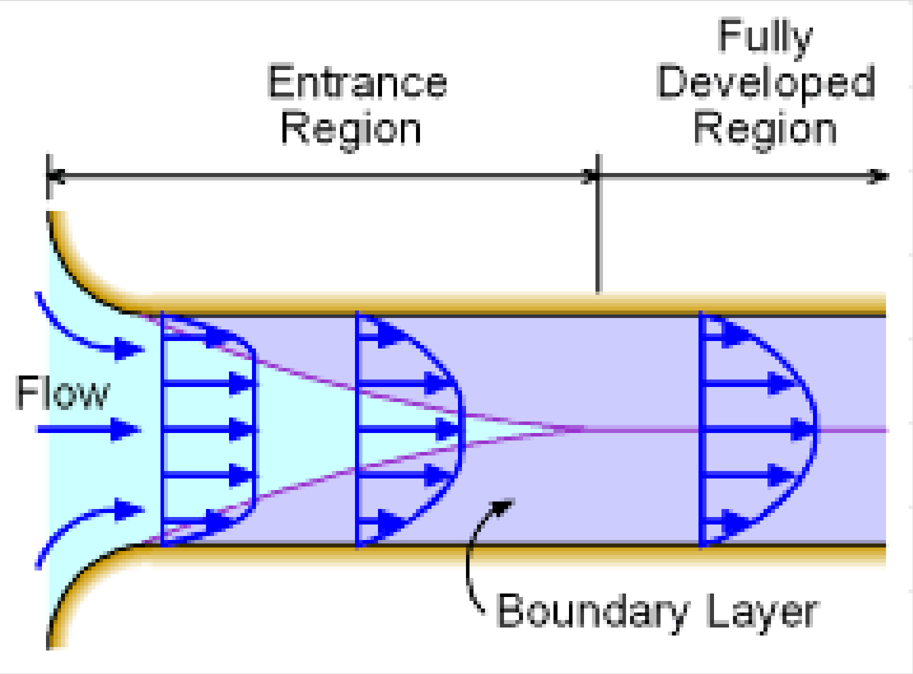


Mechanical energy flow chart for a fluid flow system that involves a pump and a turbine. Vertical dimensions show each energy term expressed as an equivalent column height of fluid, i.e., *head*.

Kinetic Energy Correction Factor, α

The kinetic energy of a fluid stream obtained from $V^2/2$ is not the same as the actual kinetic energy.

This error can be corrected by replacing the kinetic energy terms $V^2/2$ in the energy equation by $\alpha V_{avg}^2/2$, where α is the **kinetic energy correction factor**.



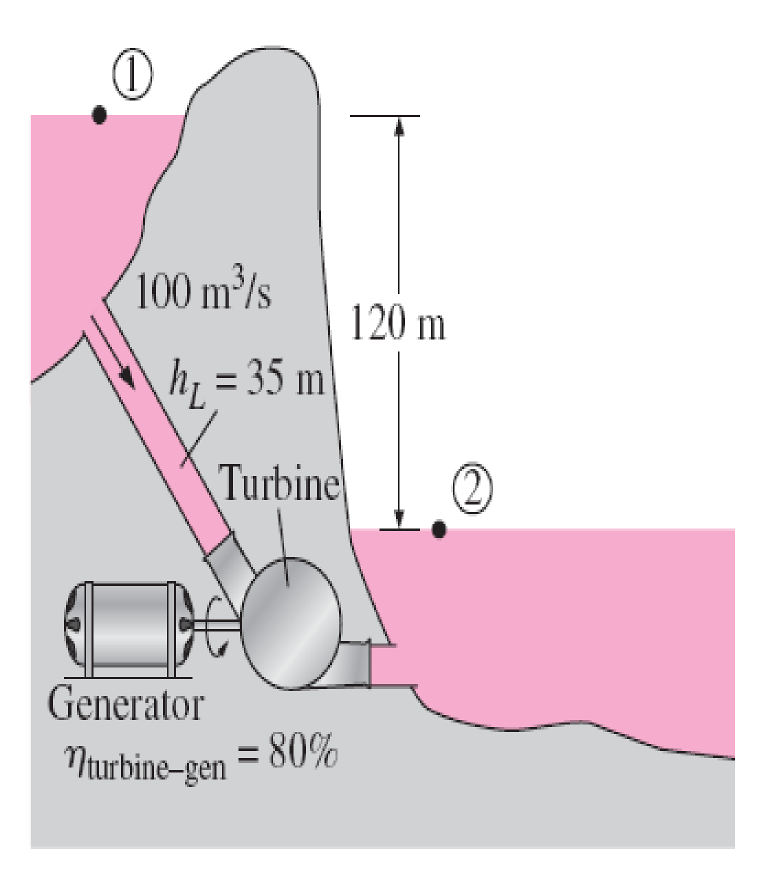
- $\alpha = 2.0$ ➔ fully developed laminar pipe flow.
- $\alpha = 1.04 \sim 1.11$ for fully developed turbulent flow in a round pipe.

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L$$

EXAMPLE 5–13

In a hydroelectric power plant, 100 m^3/s of water flows from an elevation of 120 m to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output (66.7 MW).



SOLUTION The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electric power output is to be determined.

Assumptions

1 The flow is steady and incompressible.

2 Water levels at the reservoir and the discharge site remain constant.

Analysis The mass flow rate of water through the turbine is

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(100 \text{ m}^3/\text{s}) = 10^5 \text{ kg/s}$$

We take point 2 as the reference level, and thus $z_2 = 0$. Also, both points 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the flow velocities are negligible at both points ($V_1 = V_2 = 0$). Then the energy equation for steady, incompressible flow reduces to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + h_{\text{pump},u}^{\nearrow 0} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + z_2^{\nearrow 0} + h_{\text{turbine},e} + h_L \rightarrow$$

$$h_{\text{turbine},e} = z_1 - h_L$$

Substituting, the extracted turbine head and the corresponding turbine power are

$$h_{\text{turbine},e} = z_1 - h_L = 120 - 35 = 85 \text{ m}$$

$$\dot{W}_{\text{turbine},e} = \dot{m} g h_{\text{turbine},e} = (10^5 \text{ kg/s})(9.81 \text{ m/s}^2)(85 \text{ m})\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 83{,}400 \text{ kW}$$

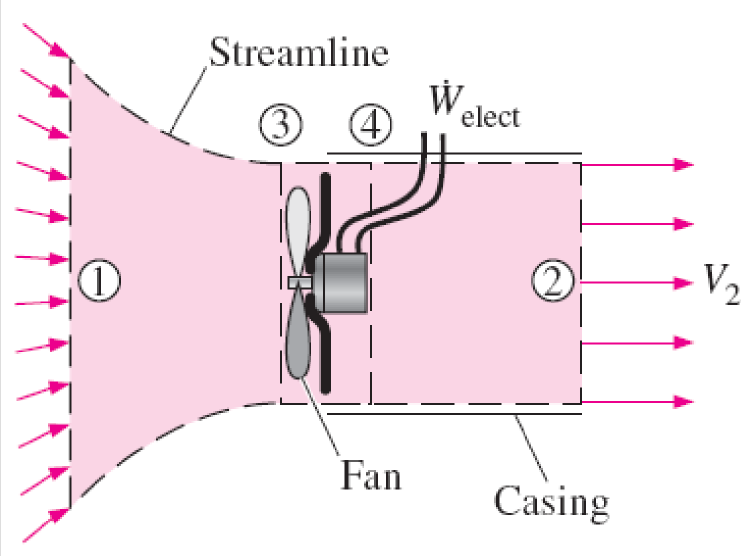
Therefore, a perfect turbine–generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{\text{electric}} = \eta_{\text{turbine-gen}} \dot{W}_{\text{turbine},e} = (0.80)(83.4 \text{ MW}) = \mathbf{66.7 \text{ MW}}$$

Discussion Note that the power generation would increase by almost 1 MW for each percentage point improvement in the efficiency of the turbine–generator unit.

EXAMPLE 5–14

A fan is to be selected to cool a computer case whose dimensions are 12 cm X 40 cm X 40 cm. Half of the volume in the case is expected to be filled with components and the other half to be air space. A 5-cm diameter hole is available at the back of the case for the installation of the fan that is to replace the air in the void spaces of the case once every second. Small low-power fan–motor combined units are available in the market and



their efficiency is estimated to be 30 %. Considering the air density to be 1.20 kg/m^3, and kinetic energy correction factor of 1.10, determine:

a) the wattage (power) of the fan–motor unit to be purchased.
b) the pressure difference across the fan.

SOLUTION A fan is to cool a computer case by completely replacing the air inside once every second.

Assumptions

1 The flow is steady and incompressible.

2 Losses other than those due to the inefficiency of the fan–motor unit are negligible ($h_L = 0$).

Analysis (*a*) Noting that half of the volume of the case is occupied by the components, the air volume in the computer case is

$$V = (\text{Void fraction})(\text{Total case volume})$$
$$= 0.5(12 \text{ cm} \times 40 \text{ cm} \times 40 \text{ cm}) = 9600 \text{ cm}^3$$

Therefore, the volume and mass flow rates of air through the case are

$$\dot{V} = \frac{V}{\Delta t} = \frac{9600 \text{ cm}^3}{1 \text{ s}} = 9600 \text{ cm}^3/\text{s} = 9.6 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\dot{m} = \rho \dot{V} = (1.20 \text{ kg/m}^3)(9.6 \times 10^{-3} \text{ m}^3/\text{s}) = 0.0115 \text{ kg/s}$$

The cross-sectional area of the opening in the case and the average air velocity through the outlet are

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.05 \text{ m})^2}{4} = 1.96 \times 10^{-3} \text{ m}^2$$

$$V = \frac{\dot{V}}{A} = \frac{9.6 \times 10^{-3} \text{ m}^3/\text{s}}{1.96 \times 10^{-3} \text{ m}^2} = 4.90 \text{ m/s}$$

We draw the control volume around the fan such that both the inlet and the outlet are at atmospheric pressure ($P_1 = P_2 = P_{atm}$), as shown in Fig. 5–56, and the inlet section 1 is large and far from the fan so that the flow velocity at the inlet section is negligible ($V_1 \cong 0$). Noting that $z_1 = z_2$ and frictional losses in flow are disregarded, the mechanical losses consist of fan losses only and the energy equation (Eq. 5–76) simplifies to

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1 \cancelto{0}{\frac{V_1^2}{2}} + \cancel{gz_1}\right) + \dot{W}_{\text{fan}} = \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2 \frac{V_2^2}{2} + \cancel{gz_2}\right) + \cancelto{0}{\dot{W}_{\text{turbine}}} + \dot{E}_{\text{mech loss, fan}}$$

Solving for $\dot{W}_{\text{fan}} - \dot{E}_{\text{mech loss, fan}} = \dot{W}_{\text{fan, }u}$ and substituting,

$$\dot{W}_{\text{fan, }u} = \dot{m}\alpha_2 \frac{V_2^2}{2} = (0.0115 \text{ kg/s})(1.10)\,\frac{(4.90 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 0.152 \text{ W}$$

Then the required electric power input to the fan is determined to be

$$\dot{W}_{\text{elect}} = \frac{\dot{W}_{\text{fan, }u}}{\eta_{\text{fan–motor}}} = \frac{0.152 \text{ W}}{0.3} = \mathbf{0.506\ W}$$

Therefore, a fan–motor rated at about a half watt is adequate for this job.
(*b*) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. This time again $z_3 = z_4$ and $V_3 = V_4$ since the fan is a narrow cross section, and the energy equation reduces to

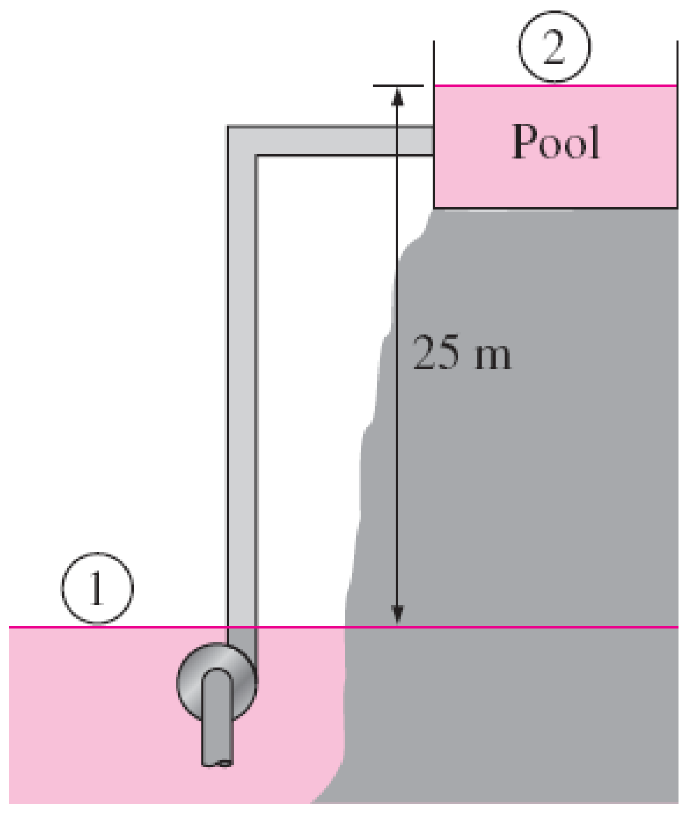
$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{\text{fan}} = \dot{m}\frac{P_4}{\rho} + \dot{E}_{\text{mech loss, fan}} \quad \rightarrow \quad \dot{W}_{\text{fan, }u} = \dot{m}\frac{P_4 - P_3}{\rho}$$

Solving for $P_4 - P_3$ and substituting,

$$P_4 - P_3 = \frac{\rho \dot{W}_{\text{fan, }u}}{\dot{m}} = \frac{(1.2 \text{ kg/m}^3)(0.152 \text{ W})}{0.0115 \text{ kg/s}}\left(\frac{1 \text{ Pa}\cdot\text{m}^3}{1 \text{ Ws}}\right) = \mathbf{15.8\ Pa}$$

EXAMPLE 5–15

Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to the water. The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m^3/s, determine the irreversible head loss of the system and the lost mechanical power during this process.



Assumptions

1 The flow is steady and incompressible.

2 The elevation difference between the reservoirs is constant.

Analysis The mass flow rate of water through the system is

$$\dot{m} = \rho \dot{V} = (1000\ \text{kg/m}^3)(0.03\ \text{m}^3/\text{s}) = 30\ \text{kg/s}$$

We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the velocities at both locations are negligible ($V_1 = V_2 = 0$). Then the energy equation for steady incompressible flow for a control volume between 1 and 2 reduces to

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1 \cancelto{0}{\frac{V_1^2}{2}} + \cancelto{0}{g z_1}\right) + \dot{W}_{\text{pump}}$$

$$= \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2 \cancelto{0}{\frac{V_2^2}{2}} + g z_2\right) + \cancelto{0}{\dot{W}_{\text{turbine}}} + \dot{E}_{\text{mech, loss}}$$

$$\dot{W}_{\text{pump}} = \dot{m} g z_2 + \dot{E}_{\text{mech, loss}} \quad \rightarrow \quad \dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump}} - \dot{m} g z_2$$

Substituting, the lost mechanical power and head loss are determined to be

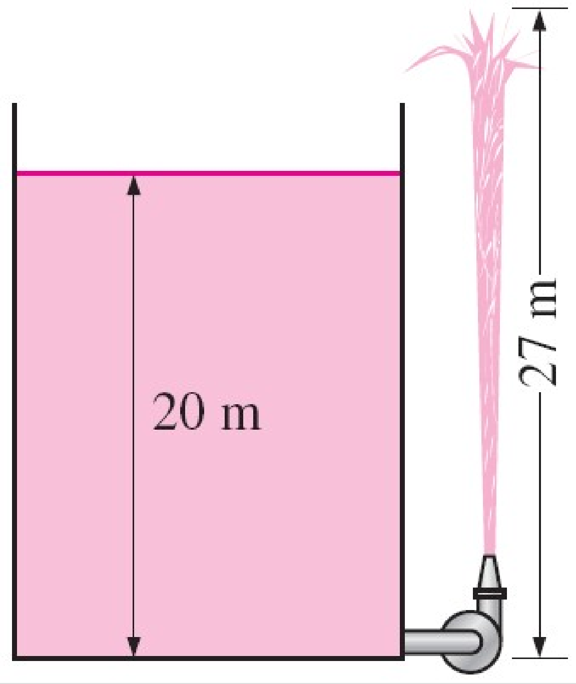
$$\dot{E}_{\text{mech, loss}} = 20\ \text{kW} - (30\ \text{kg/s})(9.81\ \text{m/s}^2)(45\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)$$

$$= \mathbf{6.76\ kW}$$

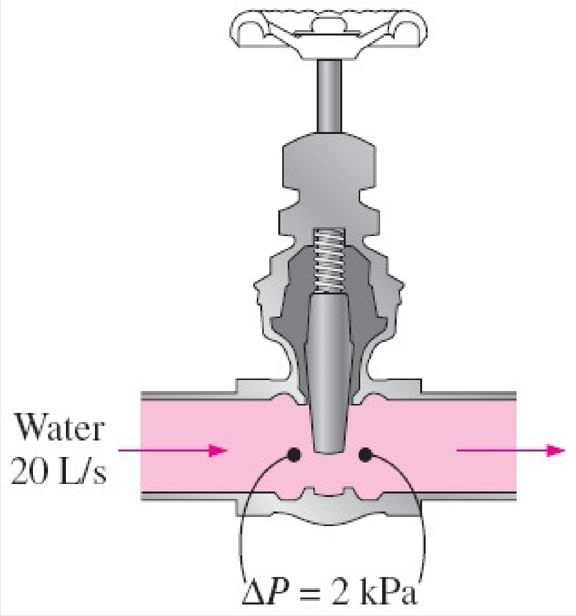
Noting that the entire mechanical losses are due to frictional losses in piping and thus $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech, loss, piping}}$, the irreversible head loss is determined to be

$$h_L = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g} = \frac{6.76\ \text{kW}}{(30\ \text{kg/s})(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)\left(\frac{1000\ \text{N}\cdot\text{m/s}}{1\ \text{kW}}\right) = \mathbf{23.0\ m}$$

5–78 The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank is at sea level, and the water surface is open to the atmosphere. In the line leading from the tank to the nozzle is a pump, which increases the pressure of water. If the water jet rises to a height of 27 m from the ground, determine the minimum pressure rise supplied by the pump to the water line.



5–81 Water flows at a rate of 20 L/s through a horizontal pipe whose diameter is constant at 3 cm. The pressure drop across a valve in the pipe is measured to be 2 kPa. Determine the irreversible head loss of the valve, and the useful pumping power needed to overcome the resulting pressure drop. (Ans: 0.204 m, 40 W)

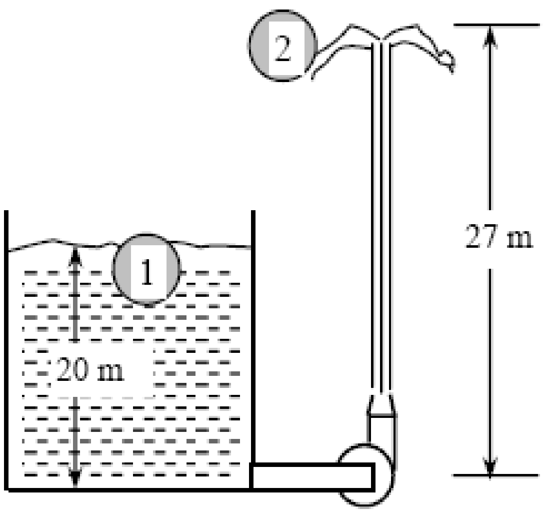


5-78

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory where $V_2 = 0$ and $P_1 = P_2 = P_{atm}$. Also, we take the reference level at the bottom of the tank. Noting that $z_1 = 20$ m and $z_2 = 27$ m, $h_L = 0$ (to get the minimum value for required pressure rise), and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the water stream reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

$$\rightarrow \quad h_{\text{pump, u}} = z_2 - z_1$$



Substituting,

$$h_{\text{pump, u}} = 27 - 20 = 7 \text{ m}$$

A water column height of 7 m corresponds to a pressure rise of

$$\Delta P_{\text{pump, min}} = \rho g h_{\text{pump, u}} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(7 \text{ m})\left(\frac{1 \text{ N}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 68.7 \text{ kN/m}^2 = \mathbf{68.7\ kPa}$$

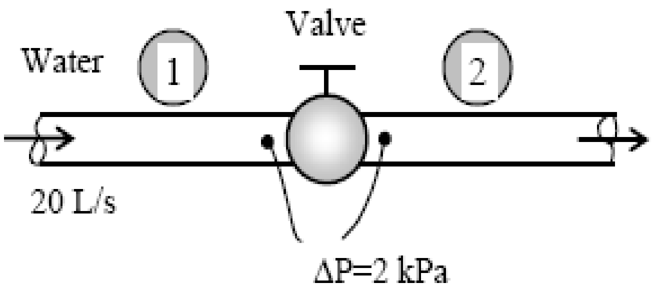
Therefore, the pump must supply a minimum pressure rise of 68.7 kPa.

5-81

Analysis We take the valve as the control volume, and points 1 and 2 at the inlet and exit of the valve, respectively. Noting that $z_1 = z_2$ and $V_1 = V_2$, the energy equation for steady incompressible flow through this control volume reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_L = \frac{P_1 - P_2}{\rho g}$$

Substituting,



$$h_L = \frac{2\ \text{kN/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right) = \mathbf{0.204\ m}$$

The useful pumping power needed to overcome this head loss is

$$\dot{W}_{\text{pump, u}} = \dot{m}gh_L = \rho\dot{V}gh_L$$

$$= (1000\ \text{kg/m}^3)(0.020\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(0.204\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{40\ W}$$

Therefore, this valve would cause a head loss of 0.204 m, and it would take 40 W of useful pumping power to overcome it.

Discussion The required useful pumping power could also be determined from

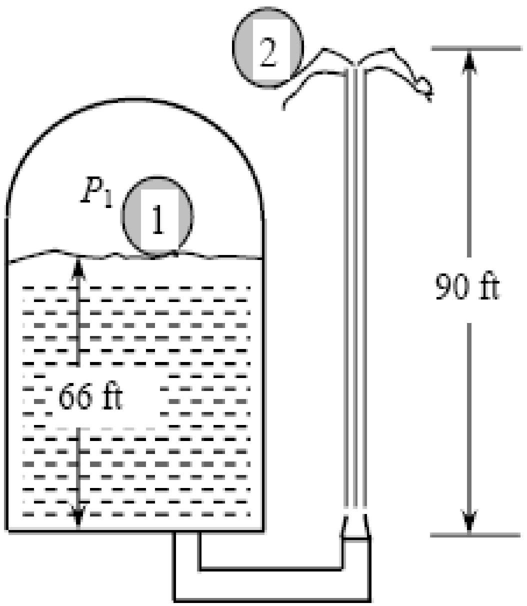
$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.020\ \text{m}^3/\text{s})(2000\ \text{Pa})\left(\frac{1\ \text{W}}{1\ \text{Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{40\ W}$$

5-82E A hose is connected to the bottom of the tank at the ground level and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, but the pressure over the water surface is unknown. Determine the minimum tank air pressure (gage) that will cause a water stream from the nozzle to rise 90 ft from the ground.

5-83 A large tank is initially filled with water 2 m above the center of a sharp-edged 10-cm-diameter orifice. The tank water surface is open to the atmosphere, and the orifice drains to the atmosphere. If the total irreversible head loss in the system is 0.3 m, determine the initial discharge velocity of water from the tank. Take the kinetic energy correction factor at the orifice to be 1.2.

5-84 Water enters a hydraulic turbine through a 30-cmdiameter pipe at a rate of 0.6 m3/s and exits through a 25-cmdiameter pipe. The pressure drop in the turbine is measured by a mercury manometer to be 1.2 m. For a combined turbine-generator efficiency of 83%, determine the net electric power output. Disregard the effect of the kinetic energy correction factors.

5-82E



Analysis We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory where $V_2 = 0$ and $P_1 = P_2 = P_{atm}$. Also, we take the reference level at the bottom of the tank. Noting that $z_1 = 66$ ft and $z_2 = 90$ ft, $h_L = 0$ (to get the minimum value for the required air pressure), and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the energy equation for steady incompressible flow through a control volume between these two points reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

or $$\frac{P_1 - P_{atm}}{\rho g} = z_2 - z_1 \rightarrow \frac{P_{1,gage}}{\rho g} = z_2 - z_1$$

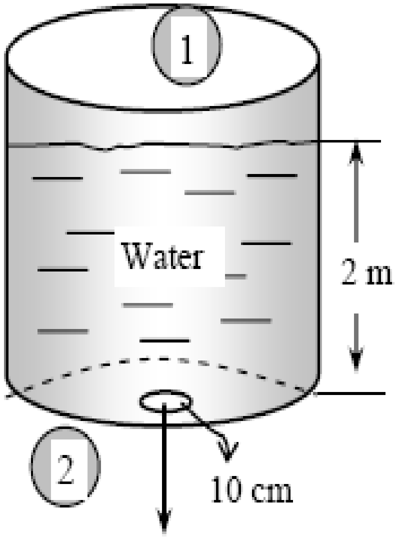
Rearranging and substituting, the gage pressure of pressurized air in the tank is determined to be

$$P_{1,\text{gage}} = \rho g(z_2 - z_1) = (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(90 - 66\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \text{psi}}{144\ \text{lbf/ft}^2}\right) = \mathbf{10.4\ psi}$$

Therefore, the gage air pressure on top of the water tank must be at least 10.4 psi.

5-83

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of the orifice. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the tank is very low $(V_1 \cong 0)$, the energy equation between these two points (in terms of heads) simplifies to



$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L$$

which yields

$$z_1 + \alpha_2 \frac{V_2^2}{2g} = z_2 + h_L$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2g(z_1 - z_2 - h_L)/\alpha} = \sqrt{2(9.81\text{ m/s}^2)(2 - 0.3\text{ m})/1.2} = \mathbf{5.27\ m/s}$$

Discussion This is the velocity that will prevail at the beginning. The mean flow velocity will decrease as the water level in the tank decreases.

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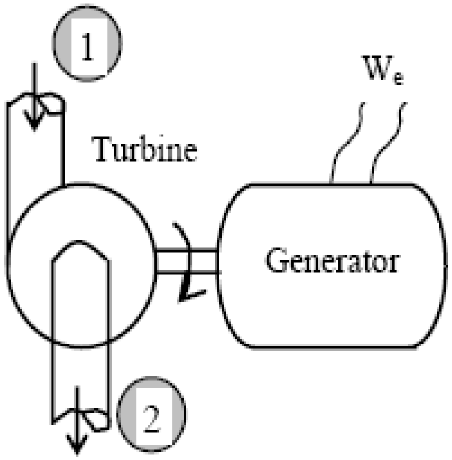
Analysis We choose points 1 and 2 at the inlet and the exit of the turbine, respectively. Noting that the elevation effects are negligible, the energy equation in terms of heads for the turbine reduces to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{turbine, e}}=\frac{P_1-P_2}{\rho_{\text{water}}g}+\frac{\alpha(V_1^2-V_2^2)}{2g} \qquad (1)$$

where



$$V_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{\pi D_1^2/4}\frac{0.6\text{ m}^3/\text{s}}{\pi(0.30\text{ m})^2/4}=8.49\text{ m/s}$$

$$V_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\pi D_2^2/4}=\frac{0.6\text{ m}^3/\text{s}}{\pi(0.25\text{ m})^2/4}=12.2\text{ m/s}$$

The pressure drop corresponding to a differential height of 1.2 m in the mercury manometer is

$$P_1-P_2=(\rho_{\text{Hg}}-\rho_{\text{water}})gh$$

$$=[(13{,}560-1000)\text{ kg/m}^3](9.81\text{ m/s}^2)(1.2\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$=148\text{ kN/m}^2=148\text{ kPa}$$

Substituting into Eq. (1), the turbine head is determined to be

$$h_{\text{turbine, e}}=\frac{148\text{ kN/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)+(1.0)\frac{(8.49\text{ m/s})^2-(12.2\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=15.1-3.9=11.2\text{ m}$$

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Then the net electric power output of this hydroelectric turbine becomes

$$\dot{W}_{\text{turbine}} = \eta_{\text{turbine-gen}} \dot{m} g h_{\text{turbine, e}} = \eta_{\text{turbine-gen}} \rho \dot{V} g h_{\text{turbine, e}}$$

$$= 0.83(1000\ \text{kg/m}^3)(0.6\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(11.2\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right) = \mathbf{55\ kW}$$

Discussion It appears that this hydroelectric turbine will generate 55 kW of electric power under given conditions. Note that almost half of the available pressure head is discarded as kinetic energy. This demonstrates the need for a larger turbine exit area and better recovery. For example, the power output can be increased to 74 kW by redesigning the turbine and making the exit diameter of the pipe equal to the inlet diameter, $D_2 = D_1$. Further, if a much larger exit diameter is used and the exit velocity is reduced to a very low level, the power generation can increase to as much as 92 kW.

Summary

- The Bernoulli Equation
  - ✓ Limitations on the Use of the Bernoulli Equation
  - ✓ Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)
  - ✓ Applications of the Bernouli Equation
- General Energy Equation
  - ✓ Energy Transfer by Heat, *Q*
  - ✓ Energy Transfer by Work, *W*
  - ✓ Shaft Work
  - ✓ Work Done by Pressure Forces
- Energy Analysis of Steady Flows
  - ✓ Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction
  - ✓ Kinetic Energy Correction Factor, α