5–1 ■ INTRODUCTION

You are already familiar with numerous **conservation laws** such as the laws of conservation of mass, conservation of energy, and conservation of momentum.

Historically, the conservation laws are first applied to a fixed quantity of matter called a *closed system* or just a *system*, and then extended to regions in space called *control volumes*.

The conservation relations are also called *balance equations* since any conserved quantity must balance during a process.

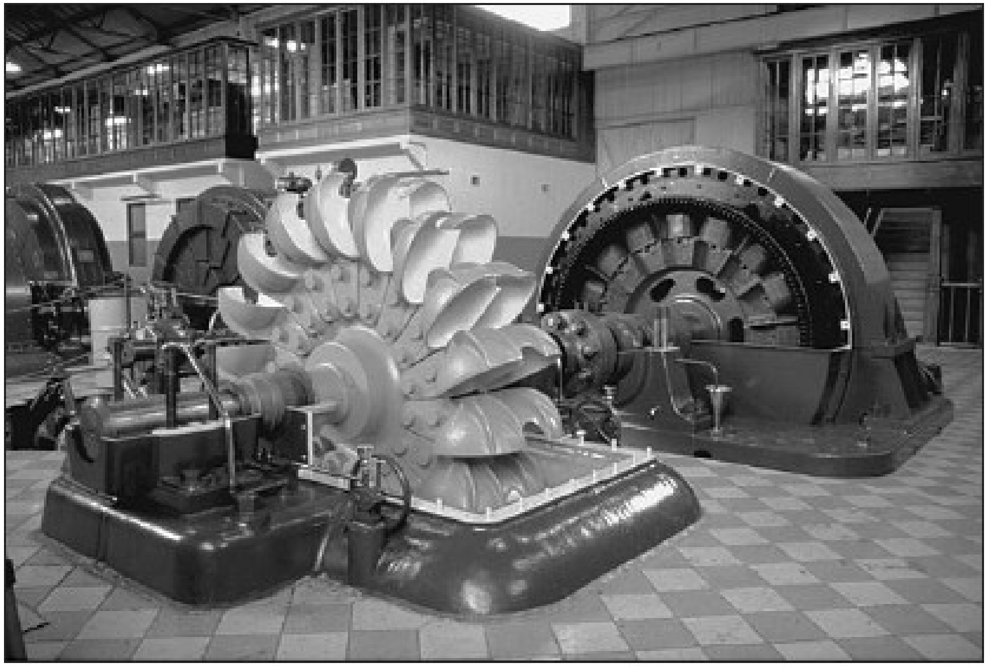


FIGURE 5–1

Many fluid flow devices such as this Pelton wheel hydraulic turbine are analyzed by applying the conservation of mass and energy principles, along with the linear momentum equation.

Conservation of Mass

The conservation of mass relation for a closed system undergoing a change is expressed as m_{sys} = constant or $dm_{sys}/dt = 0$, which is the statement that the mass of the system remains constant during a process.

Mass balance for a control volume (CV) in rate form:

Conservation of mass: $$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt}$$

$\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ the total rates of mass flow into and out of the control volume

dm_{CV}/dt the rate of change of mass within the control volume boundaries.

Continuity equation: In fluid mechanics, the conservation of mass relation written for a differential control volume is usually called the *continuity equation*.

The Linear Momentum Equation

Linear momentum: The product of the mass and the velocity of a body is called the *linear momentum* or just the *momentum* of the body.

The momentum of a rigid body of mass *m* moving with a velocity $\vec{V}$ is $m\vec{V}$.

Newton's second law: The acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body.

Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero, and thus the momentum of such systems is conserved.

Linear momentum equation: In fluid mechanics, Newton's second law is usually referred to as the *linear momentum equation.*

Conservation of Energy

The conservation of energy principle (the energy balance): The net energy transfer to or from a system during a process be equal to the change in the energy content of the system.

Energy can be transferred to or from a closed system by heat or work.

Control volumes also involve energy transfer via mass flow.

Conservation of energy: $$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{CV}}}{dt}$$

$\dot{E}_{\text{in}}$ and $\dot{E}_{\text{out}}$ the total rates of energy transfer into and out of the control volume

dE_{CV}/dt the rate of change of energy within the control volume boundaries

In fluid mechanics, we usually limit our consideration to mechanical forms of energy only.

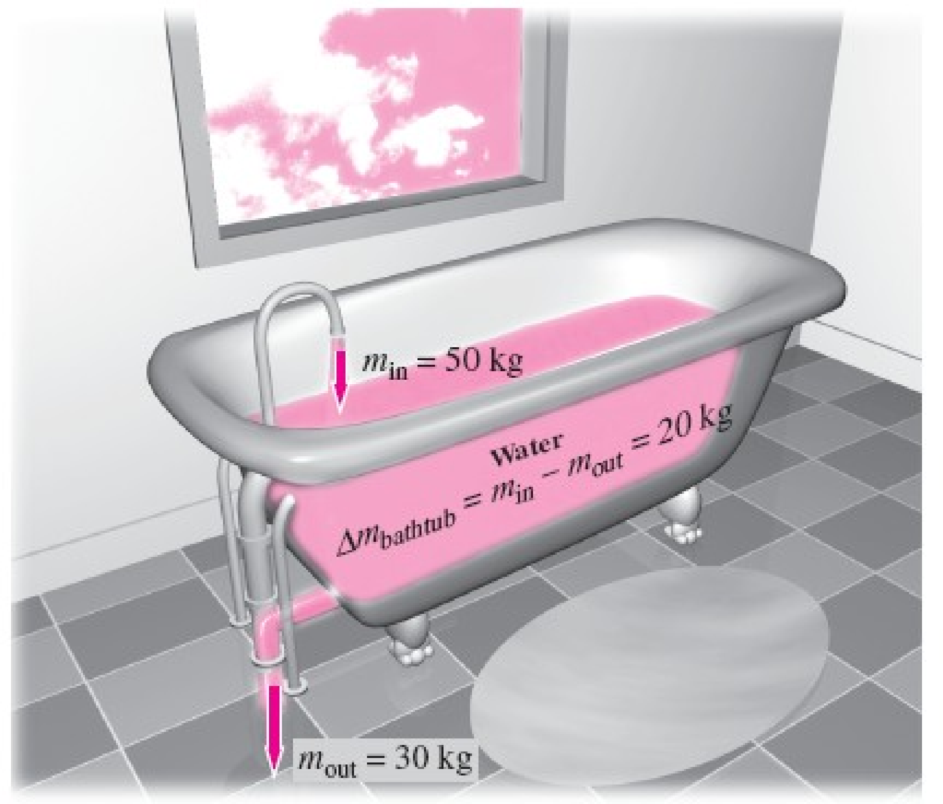
Conservation of Mass Principle

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.

$$\left(\begin{array}{c}\text{Total mass entering} \\ \text{the CV during } \Delta t\end{array}\right) - \left(\begin{array}{c}\text{Total mass leaving} \\ \text{the CV during } \Delta t\end{array}\right) = \left(\begin{array}{c}\text{Net change of mass} \\ \text{within the CV during } \Delta t\end{array}\right)$$

$$m_{in} - m_{out} = \Delta m_{CV} \quad \text{(kg)}$$

$$\dot{m}_{in} - \dot{m}_{out} = dm_{CV}/dt \quad \text{(kg/s)}$$



$\dot{m}_{in}$ and $\dot{m}_{out}$ the total rates of mass flow into and out of the control volume

dm_{CV}/dt the rate of change of mass within the control volume boundaries.

Mass balance is applicable to any control volume undergoing any kind of process.

Conservation of mass principle for an ordinary bathtub.

$$dm = \rho \, dV$$

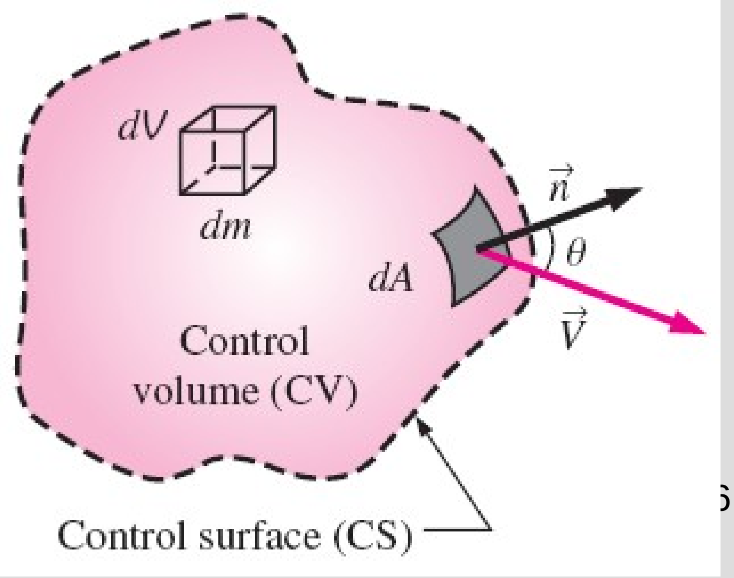
Total mass within the CV: $$m_{CV} = \int_{CV} \rho \, dV$$

Rate of change of mass within the CV: $$\frac{dm_{CV}}{dt} = \frac{d}{dt} \int_{CV} \rho \, dV$$

Normal component of velocity: $$V_n = V \cos \theta = \vec{V} \cdot \vec{n}$$

Differential mass flow rate: $$\delta \dot{m} = \rho V_n \, dA = \rho (V \cos \theta) \, dA = \rho (\vec{V} \cdot \vec{n}) \, dA$$

Net mass flow rate: $$\dot{m}_{net} = \int_{CS} \delta \dot{m} = \int_{CS} \rho V_n \, dA = \int_{CS} \rho (\vec{V} \cdot \vec{n}) \, dA$$



The differential control volume *dV* and the differential control surface *dA* used in the derivation of the conservation of mass relation.

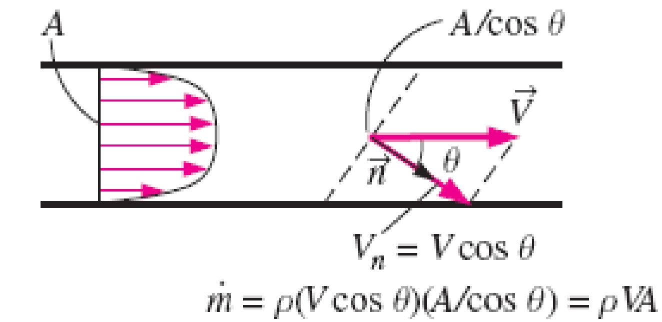
General conservation of mass:
$$\frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

The time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

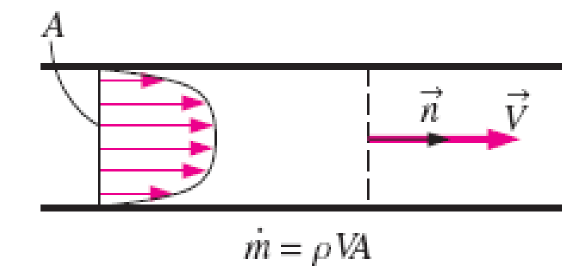
$$\frac{d}{dt}\int_{CV} \rho \, dV + \sum_{\text{out}} \rho |V_n| A - \sum_{\text{in}} \rho |V_n| A = 0$$

$$\frac{d}{dt}\int_{CV} \rho \, dV = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

$$\frac{dm_{CV}}{dt} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$



(*a*) Control surface *at an angle* to the flow



(*b*) Control surface *normal* to the flow

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b(\vec{V}\cdot\vec{n})\, dA$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$B = m \qquad\qquad b = 1 \qquad\qquad b = 1$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\frac{dm_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA$$

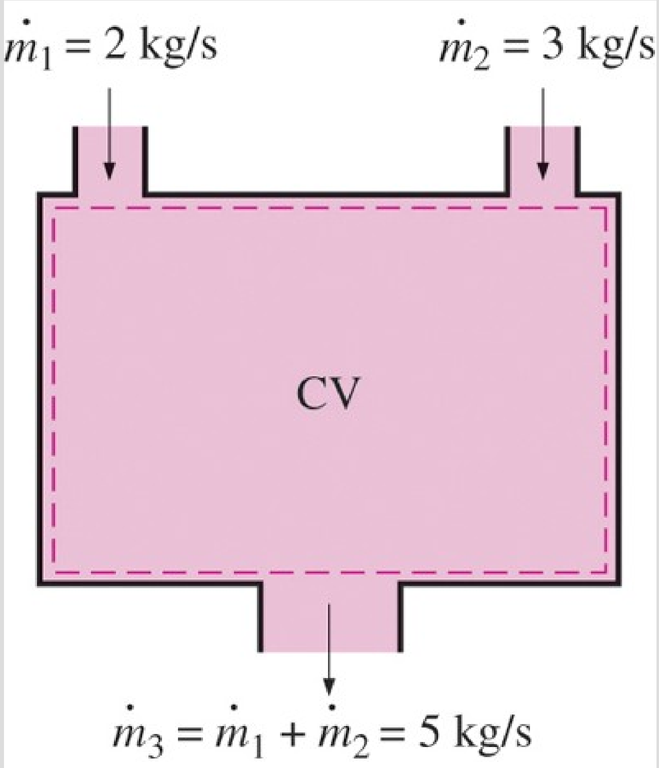
The conservation of mass equation is obtained by replacing *B* in the Reynolds transport theorem by mass *m*, and *b* by 1 (*m* per unit mass = *m*/*m* = 1).

A control surface should always be selected *normal to the flow* at all locations where it crosses the fluid flow to avoid complications, even though the result is the same.

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.



Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate.*

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad (\text{kg/s})$$

Multiple inlets and exits

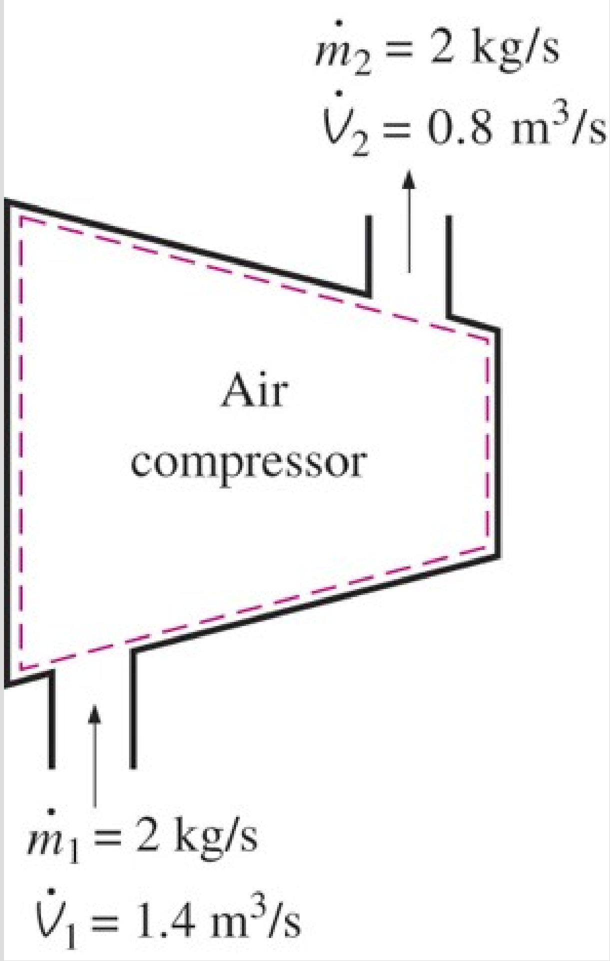
$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

Special Case: Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.



$$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \qquad (\text{m}^3/\text{s})$$

Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow (single stream)

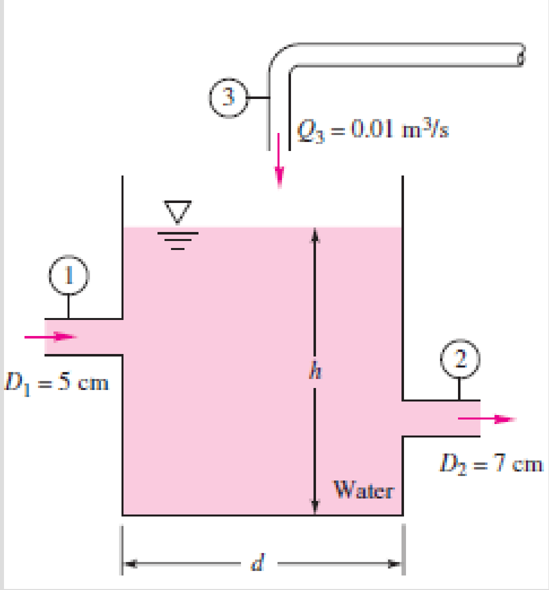
There is no such thing as a "conservation of volume" principle.

However, for steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

Example 1

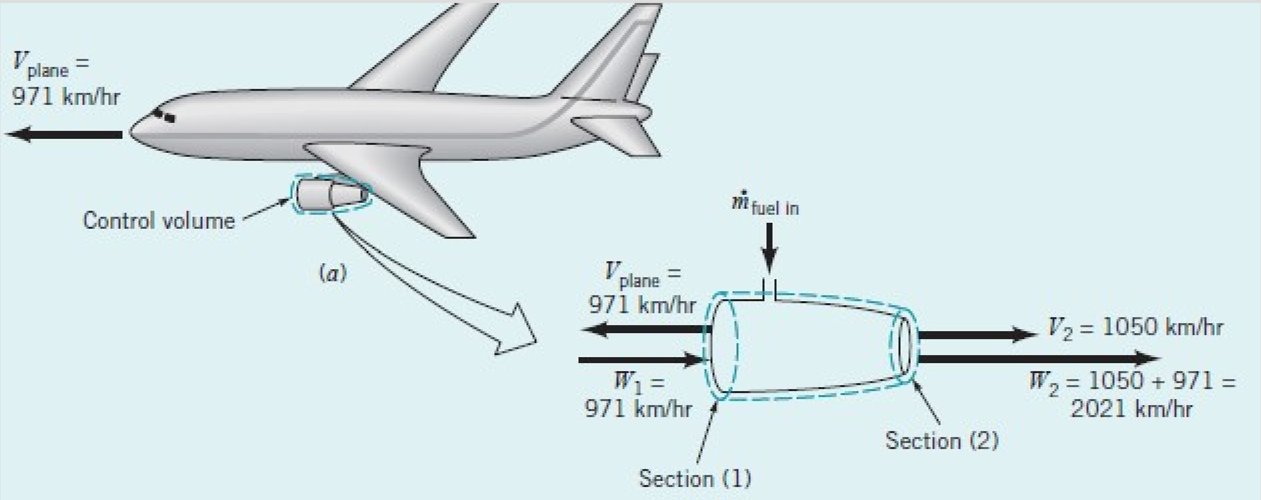
The open tank in Fig. P3.14 contains water at 20°C and is being filled through section 1. Assume incompressible flow. First derive an analytic expression for the water-level change *dh/dt* in terms of arbitrary volume flows (Q_1, Q_2, Q_3) and tank diameter d. Then, if the water level h is constant, determine the exit velocity V_2 for the given data $V_1 = 3$ m/s and $Q_3 = 0.01$ m^3/s.



Example 2

GIVEN An airplane moves forward at a speed of 971 km/hr as shown in Fig. E5.6a. The frontal intake area of the jet engine is 0.80 m^2 and the entering air density is 0.736 kg/m^3. A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m^2, and the exhaust gas density is 0.515 kg/m^3.

FIND Estimate the mass flowrate of fuel into the engine in kg/hr.



Linear Momentum

$$\sum \vec{F} = \frac{d}{dt}\int_{\mathrm{CV}} \rho \vec{V}\, dV + \int_{\mathrm{CS}} \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA$$

Momentum-Flux Correction Factor

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \sum_{out} \beta \dot{m} \vec{V}_{avg} - \sum_{in} \beta \dot{m} \vec{V}_{avg}$$

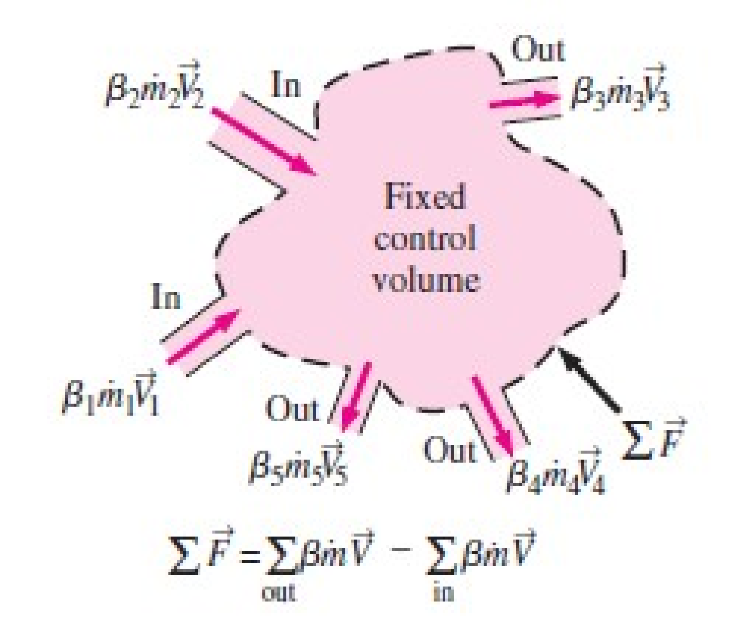
Momentum flux across an inlet or outlet: $\displaystyle\int_{A_c} \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA_c = \beta \dot{m} \vec{V}_{avg}$

$$\beta = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\dot{m} V_{avg}} = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\rho V_{avg} A_c V_{avg}}$$

Momentum-flux correction factor: $\displaystyle\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{avg}}\right)^2 dA_c$

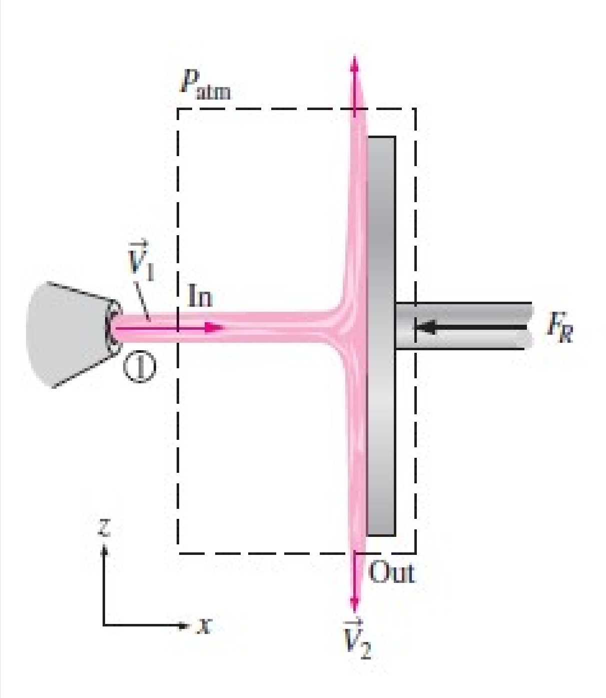
Steady Flow

Steady linear momentum equation: $$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$



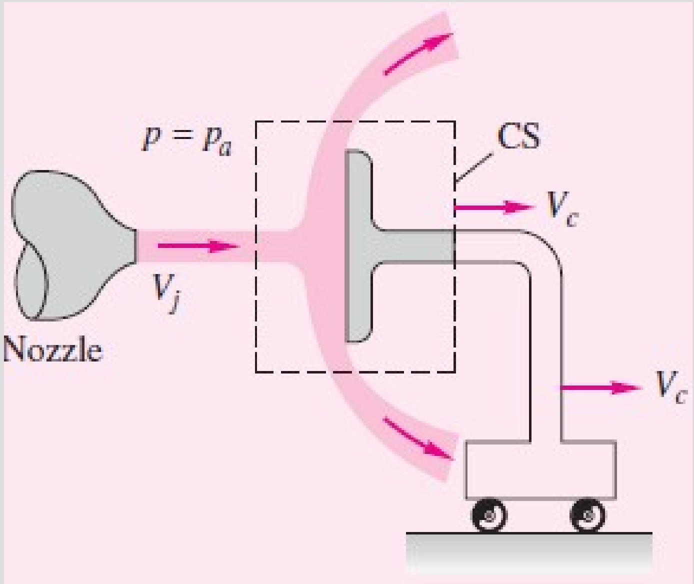
Example

Water is accelerated by a nozzle to an average speed of 20 m/s, and strikes a stationary vertical plate at a rate of 10 kg/s with a normal velocity of 20 m/s (Fig. 6–22). After the strike, the water stream splatters off in all directions in the plane of the plate. Determine the force needed to prevent the plate from moving horizontally due to the water stream.



Example

A water jet of velocity V_j impinges normal to a flat plate that moves to the right at velocity V_c, as shown in Fig. 3.9*a*. Find the force required to keep the plate moving at constant velocity if the jet density is 1000 kg/m^3, the jet area is 3 cm^2, and V_j and V_c are 20 and 15 m/s, respectively. Neglect the weight of the jet and plate, and assume steady flow with respect to the moving plate with the jet splitting into an equal upward and downward half-jet.



The suggested control volume in Fig. 3.9*a* cuts through the plate support to expose the desired forces R_x and R_y. This control volume moves at speed V_c and thus is fixed relative to the plate, as in Fig. 3.9*b*. We must satisfy both mass and momentum conservation for the assumed steady flow pattern in Fig. 3.9*b*. There are two outlets and one inlet, and Eq. (3.30) applies for mass conservation:

$$\dot{m}_{out} = \dot{m}_{in}$$

or

$$\rho_1 A_1 V_1 + \rho_2 A_2 V_2 = \rho_j A_j (V_j - V_c) \tag{1}$$

We assume that the water is incompressible $\rho_1 = \rho_2 = \rho_j$, and we are given that $A_1 = A_2 = \frac{1}{2}A_j$. Therefore Eq. (1) reduces to

$$V_1 + V_2 = 2(V_j - V_c) \tag{2}$$

Strictly speaking, this is all that mass conservation tells us. However, from the symmetry of the jet deflection and the neglect of gravity on the fluid trajectory, we conclude that the two velocities V_1 and V_2 must be equal, and hence Eq. (2) becomes

$$V_1 = V_2 = V_j - V_c \tag{3}$$

This equality can also be predicted by Bernoulli's equation in Sect 3.5. For the given numerical values, we have

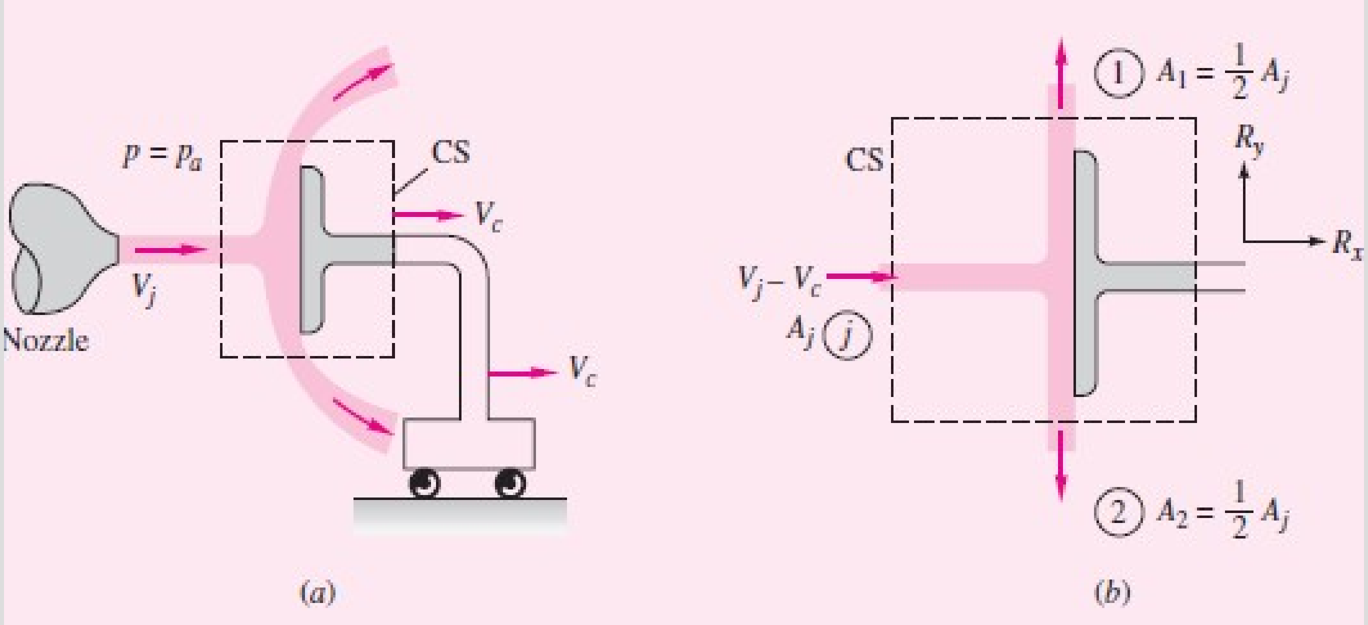
$$V_1 = V_2 = 20 - 15 = 5 \text{ m/s}$$

Now we can compute R_x and R_y from the two components of momentum conservation. Equation (3.40) applies with the unsteady term zero:

$$\sum F_x = R_x = \dot{m}_1 u_1 + \dot{m}_2 u_2 - \dot{m}_j u_j \tag{4}$$

where from the mass analysis, $\dot{m}_1 = \dot{m}_2 = \frac{1}{2}\dot{m}_j = \frac{1}{2}\rho_j A_j (V_j - V_c)$. Now check the flow directions at each section: $u_1 = u_2 = 0$, and $u_j = V_j - V_c = 5$ m/s. Thus Eq. (4) becomes

$$R_x = -\dot{m}_j u_j = -[\rho_j A_j (V_j - V_c)](V_j - V_c) \tag{5}$$

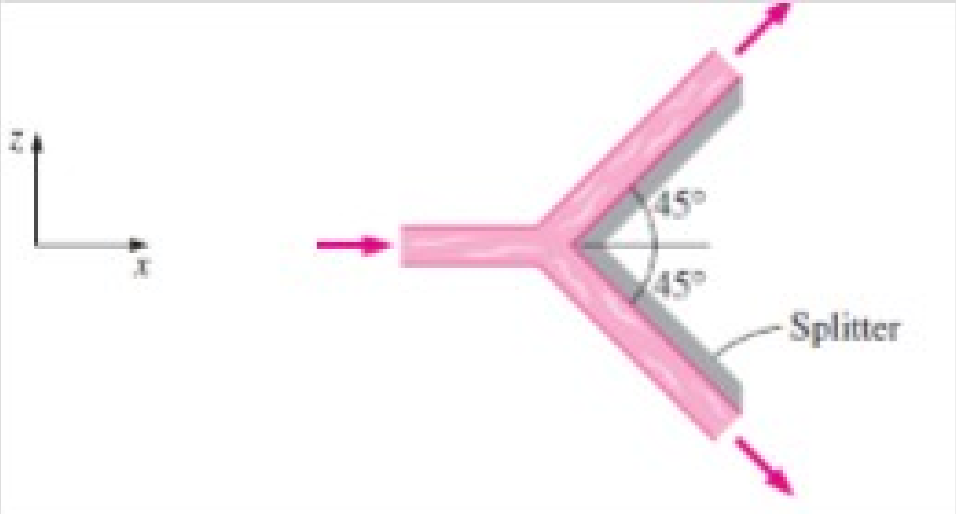


For the given numerical values we have

$$R_x = -(1000 \text{ kg/m}^3)(0.0003 \text{ m}^2)(5 \text{ m/s})^2 = -7.5 \text{ (kg} \cdot \text{m)/s}^2 = -7.5 \text{ N} \qquad \textit{Ans.}$$

Problem

- A 2.8 m3/s water jet is moving in the positive *x*-direction at 5.5 m/s. The stream hits a stationary splitter, such that half of the flow is diverted upward at 45° and the other half is directed downward, and both streams have a final speed of 5.5 m/s. Disregarding gravitational effects, determine the *x*- and *z*-components of the force required to hold the splitter in place against the water force.

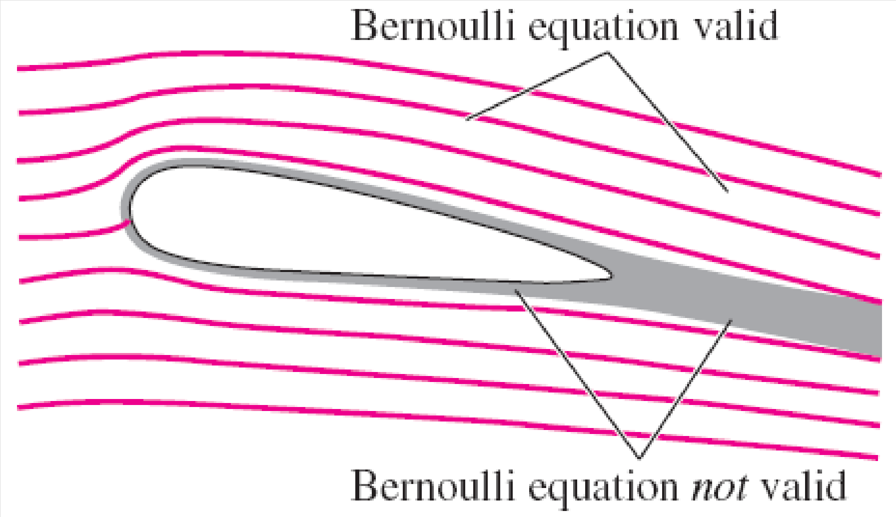


5–4 ■ THE BERNOULLI EQUATION

Bernoulli equation: An approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.

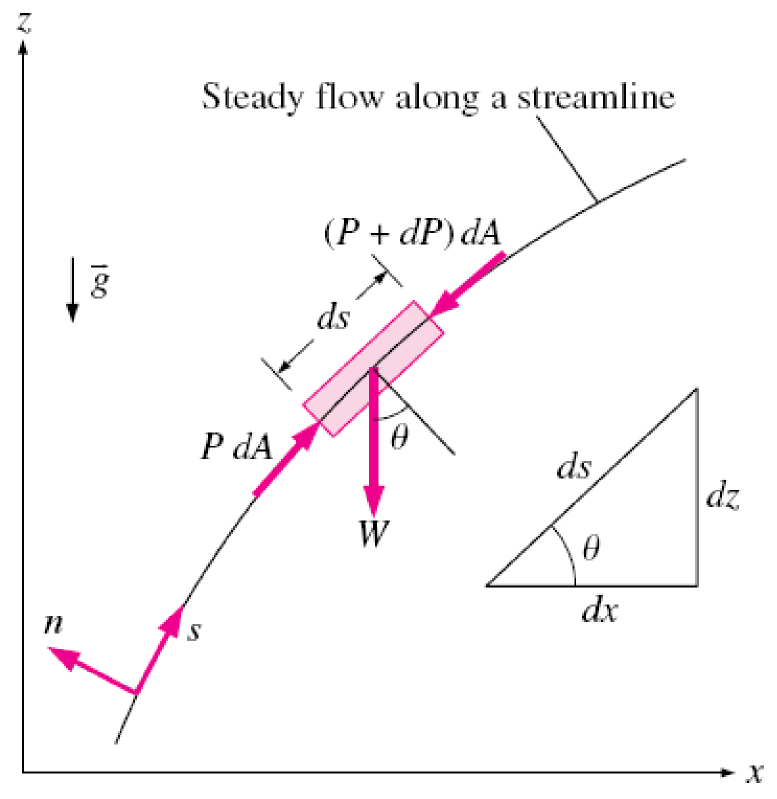
Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.

The Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.



The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes*.

Derivation of the Bernoulli Equation



The forces acting on a fluid particle along a streamline.

$$\sum F_s = ma_s \qquad P\,dA - (P + dP)\,dA - W \sin\theta = mV\frac{dV}{ds}$$

$$m = \rho V = \rho\, dA\, ds \qquad W = mg = \rho g\, dA\, ds$$

$$\sin\theta = dz/ds \qquad -dP\,dA - \rho g\, dA\, ds\frac{dz}{ds} = \rho\, dA\, ds\, V\frac{dV}{ds}$$

$$-dP - \rho g\, dz = \rho V\, dV \qquad V\,dV = \tfrac{1}{2}\, d(V^2)$$

$$\frac{dP}{\rho} + \tfrac{1}{2}\, d(V^2) + g\, dz = 0$$

Steady flow:

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

Steady, incompressible flow:

Bernoulli equation

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.

The Bernoulli equation between any two points on the same streamline:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

(Steady flow along a streamline)

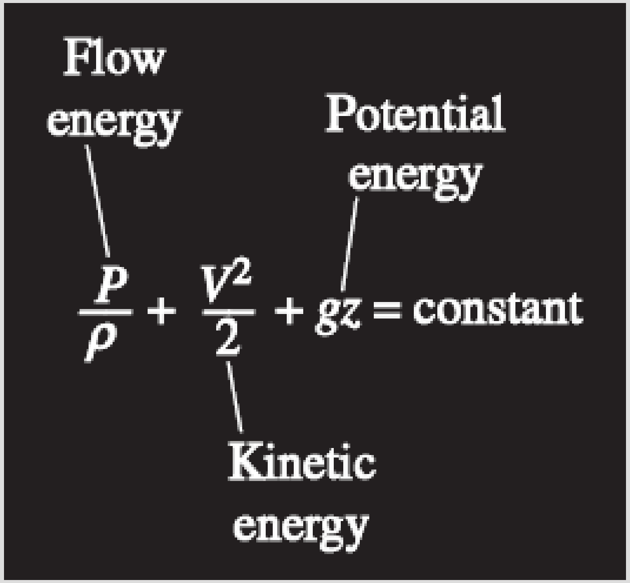
General:

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Incompressible flow (ρ = constant):

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

The incompressible Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.

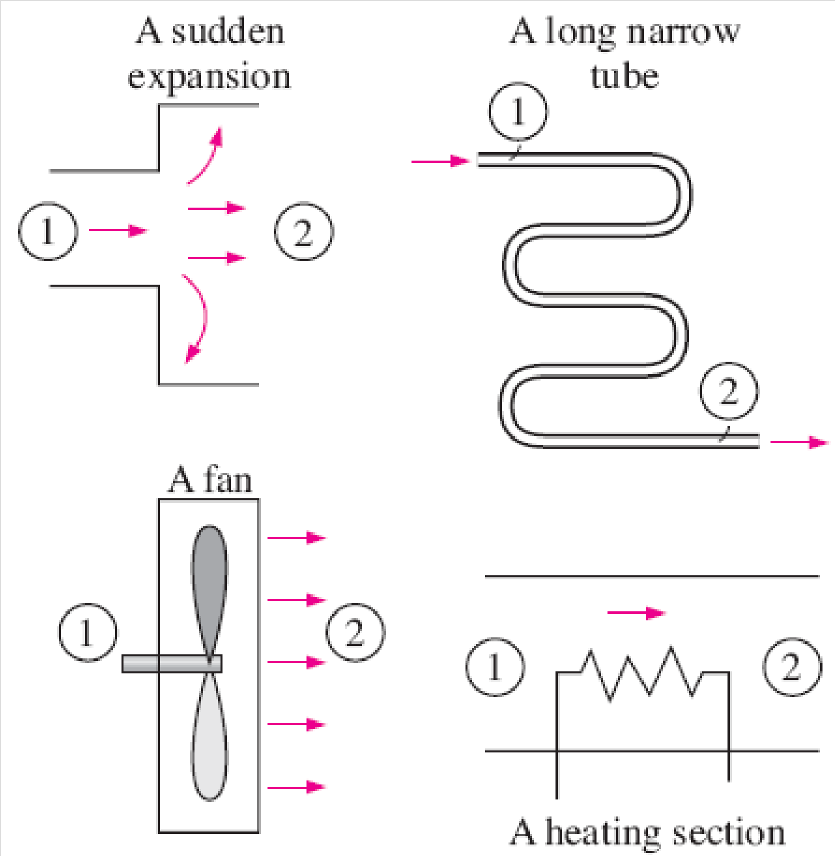


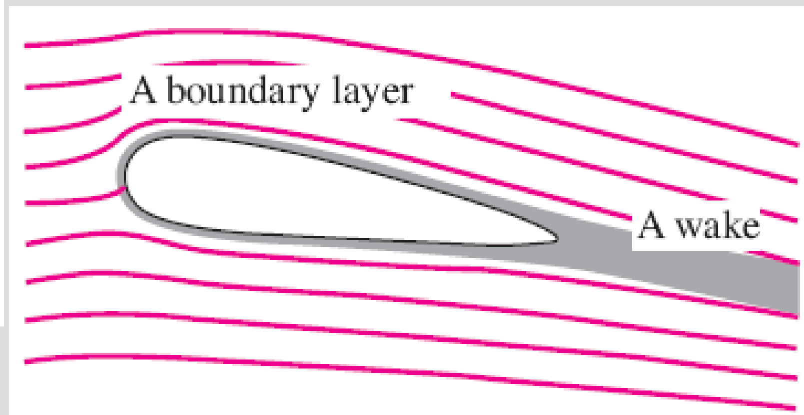
The Bernoulli equation states that the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow.

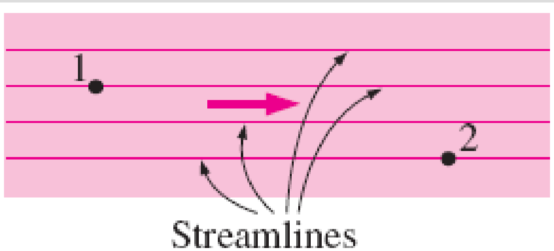
- The Bernoulli equation can be viewed as the "*conservation of mechanical energy principle*."
- This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately.
- The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.
- There is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.
- The Bernoulli equation is commonly used in practice since a variety of practical fluid flow problems can be analyzed to reasonable accuracy with it.

Limitations on the Use of the Bernoulli Equation

1. **Steady flow** The Bernoulli equation is applicable to *steady flow*.
2. **Frictionless flow** Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible.
3. **No shaft work** The Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles. When these devices exist, the energy equation should be used instead.
4. **Incompressible flow** Density is taken constant in the derivation of the Bernoulli equation. The flow is incompressible for liquids and also by gases at Mach numbers less than about 0.3.
5. **No heat transfer** The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
6. **Flow along a streamline** Strictly speaking, the Bernoulli equation is applicable along a streamline. However, when a region of the flow is *irrotational* and there is negligibly small *vorticity* in the flow field, the Bernoulli equation becomes applicable *across* streamlines as well.







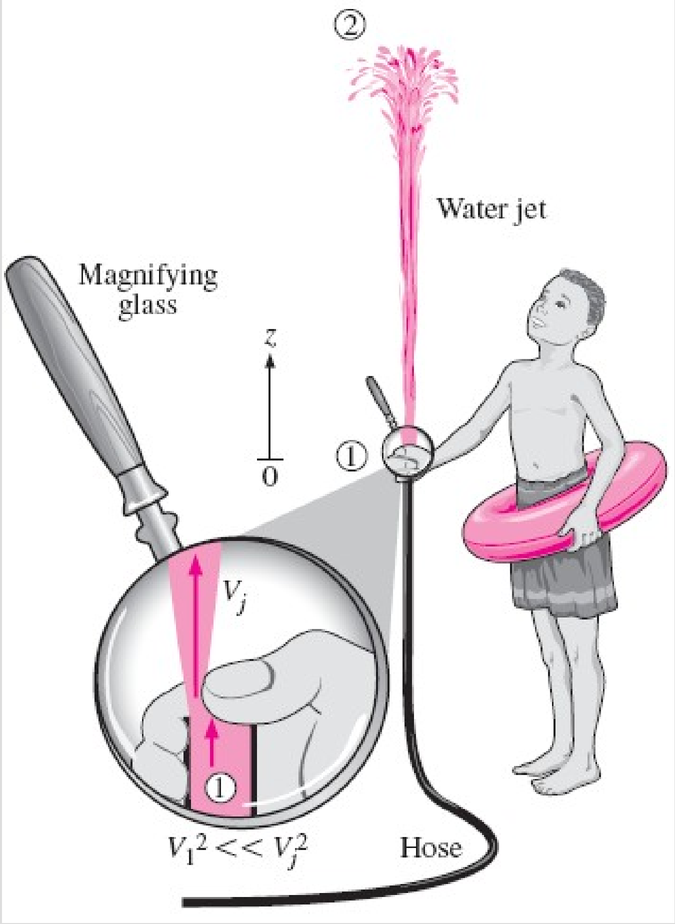
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Frictional effects, heat transfer, and components that disturb the streamlined structure of flow make the Bernoulli equation invalid. It should *not* be used in any of the flows shown here.

When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).

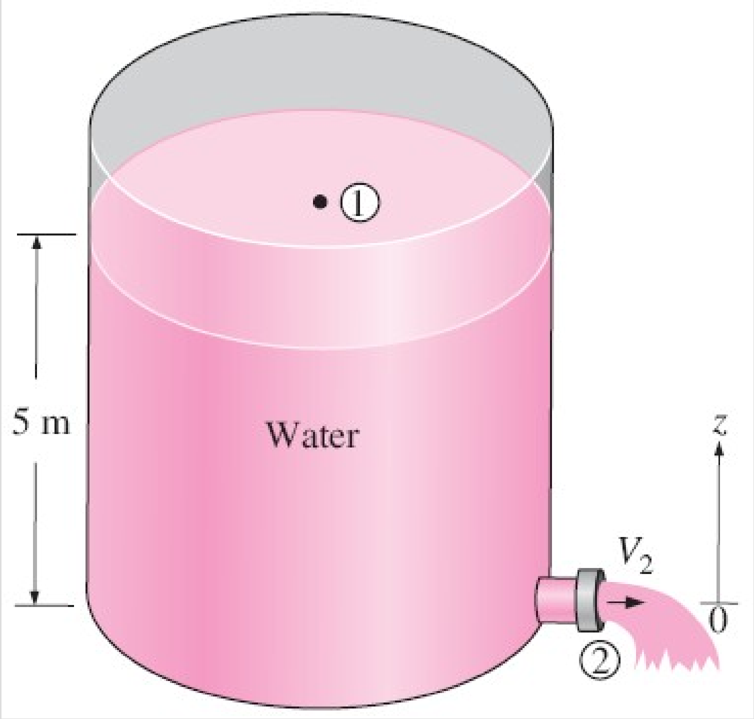
EXAMPLE 5–5 Spraying Water into the Air

Water is flowing from a hose attached to a water main at 400 kPa gage (Fig. 5–38). A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. If the hose is held upward, what is the maximum height that the jet could achieve?

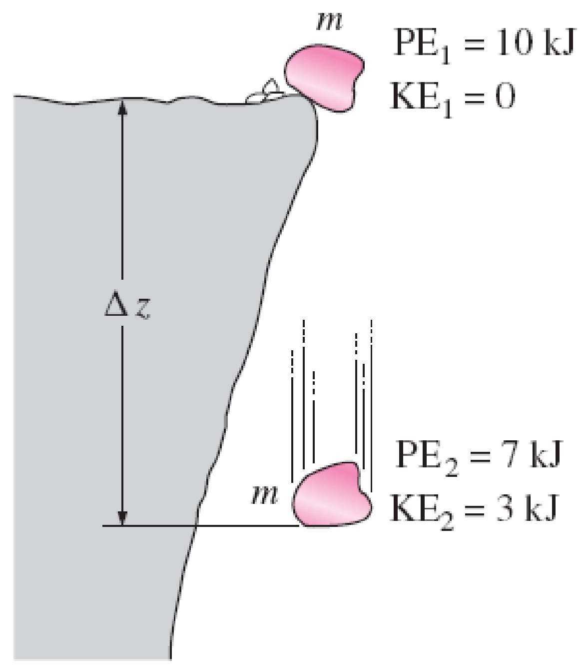


EXAMPLE 5–6 Water Discharge from a Large Tank

A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap (Fig. 5–39). A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the water velocity at the outlet.



5–5 ■ GENERAL ENERGY EQUATION



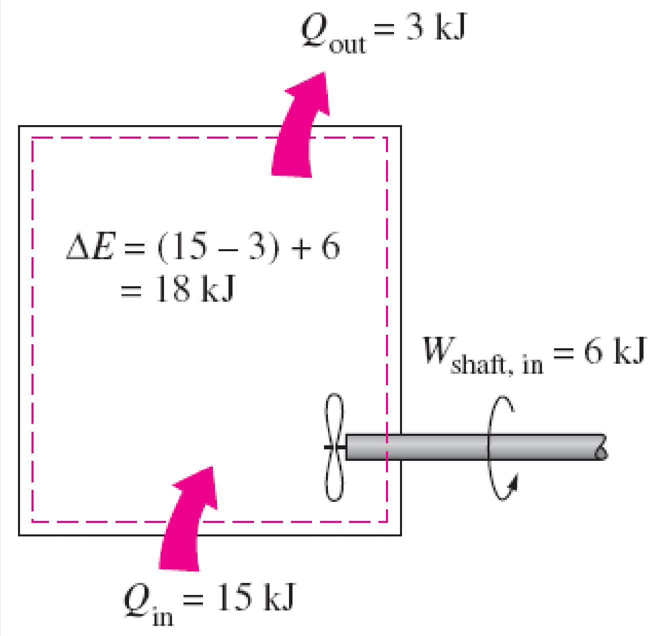
The first law of thermodynamics (the conservation of energy principle): Energy cannot be created or destroyed during a process; it can only change forms.

$$E_{in} - E_{out} = \Delta E$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{dE_{sys}}{dt} \qquad \dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{sys} \rho e \, dV$$

$$\dot{Q}_{\text{net in}} = \dot{Q}_{in} - \dot{Q}_{out} \qquad \dot{W}_{\text{net in}} = \dot{W}_{in} - \dot{W}_{out}$$

$$e = u + \text{ke} + \text{pe} = u + \frac{V^2}{2} + gz$$



The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.

Energy Transfer by Heat, *Q*

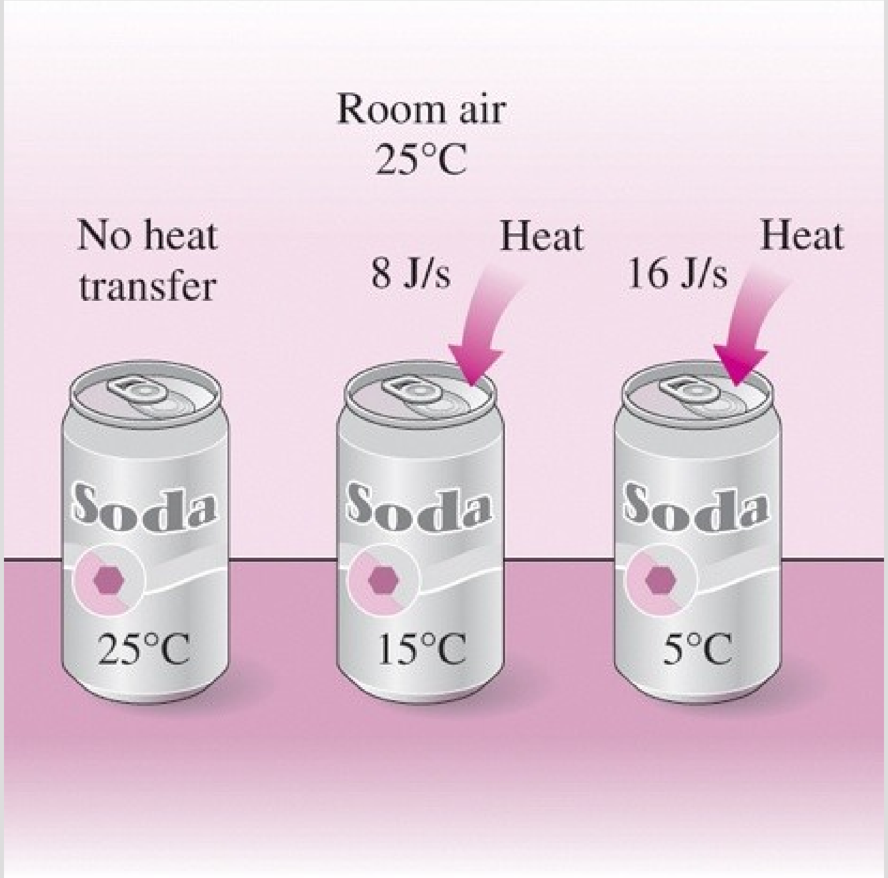
Thermal energy: The sensible and latent forms of internal energy.

Heat Transfer: The transfer of energy from one system to another as a result of a temperature difference.

The direction of heat transfer is always from the higher-temperature body to the lower-temperature one.

Adiabatic process: A process during which there is no heat transfer.

Heat transfer rate: The time rate of heat transfer.



Temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer.

Energy Transfer by Work, *W*

- **Work**: The energy transfer associated with a force acting through a distance.
- A rising piston, a rotating shaft, and an electric wire crossing the system boundaries are all associated with work interactions.
- **Power:** The time rate of doing work.
- Car engines and hydraulic, steam, and gas turbines produce work; compressors, pumps, fans, and mixers consume work.

$$W_{\text{total}} = W_{\text{shaft}} + W_{\text{pressure}} + W_{\text{viscous}} + W_{\text{other}}$$

W_{shaft} The work transmitted by a rotating shaft

$W_{pressure}$ The work done by the pressure forces on the control surface

$W_{viscous}$ The work done by the normal and shear components of viscous forces on the control surface

W_{other} The work done by other forces such as electric, magnetic, and surface tension

Shaft Work

A force F acting through a moment arm r generates a torque T

$$\mathrm{T} = Fr \quad \rightarrow \quad F = \frac{\mathrm{T}}{r}$$

This force acts through a distance s

$$s = (2\pi r)n$$

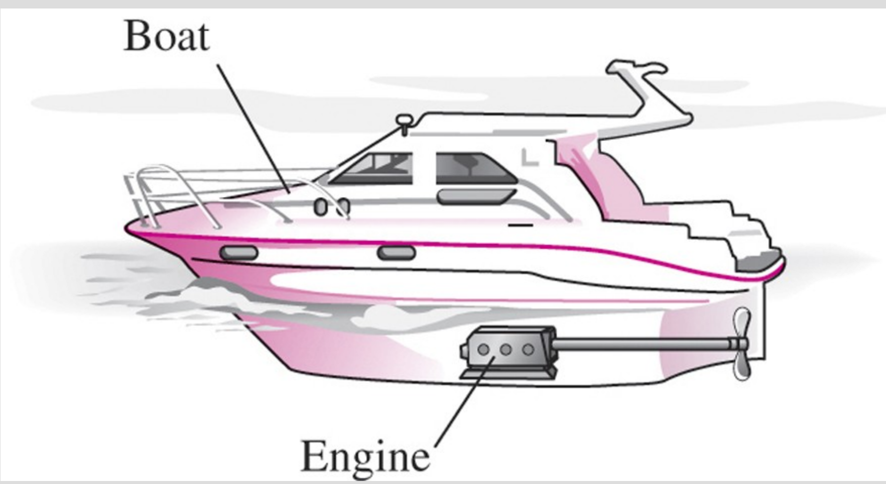
Shaft work

$$W_{sh} = Fs = \left(\frac{\mathrm{T}}{r}\right)(2\pi rn) = 2\pi n\mathrm{T} \qquad (\mathrm{kJ})$$

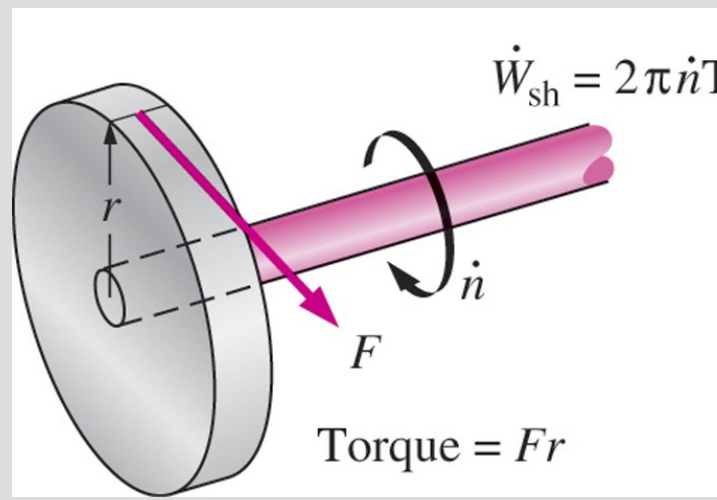
The power transmitted through the shaft is the shaft work done per unit time:

$$\dot{W}_{shaft} = \omega \mathrm{T}_{shaft} = 2\pi\dot{n}\mathrm{T}_{shaft}$$

$$\dot{W}_{sh} = 2\pi\dot{n}\mathrm{T} \qquad (\mathrm{kW})$$



Energy transmission through rotating shafts is commonly encountered in practice.



Shaft work is proportional to the torque applied and the number of revolutions of the shaft.

Work Done by Pressure Forces

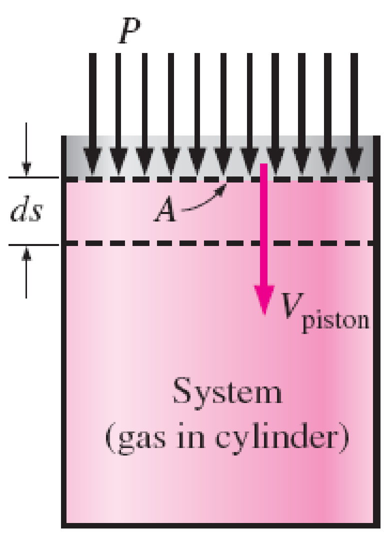
$$\delta W_{\text{boundary}} = PA\ ds$$

$$\delta \dot{W}_{\text{pressure}} = \delta \dot{W}_{\text{boundary}} = PAV_{\text{piston}} \qquad V_{\text{piston}} = ds/dt$$

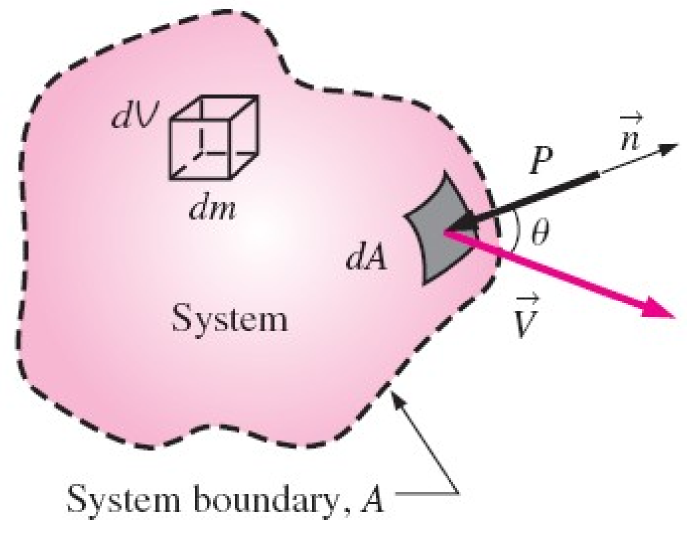
$$\delta \dot{W}_{\text{pressure}} = -P\ dA\ V_n = -P\ dA(\vec{V} \cdot \vec{n})$$

$$\dot{W}_{\text{pressure, net in}} = -\int_A P(\vec{V} \cdot \vec{n})\ dA = -\int_A \frac{P}{\rho}\rho(\vec{V} \cdot \vec{n})\ dA$$

$$\dot{W}_{\text{net in}} = \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \dot{W}_{\text{shaft, net in}} - \int_A P(\vec{V} \cdot \vec{n})\ dA$$



(a)



(b)

The pressure force acting on (*a*) the moving boundary of a system in a piston-cylinder device, and (*b*) the differential surface area of a system of arbitrary shape.

The conservation of energy equation is obtained by replacing *B* in the Reynolds transport theorem by energy *E* and *b* by *e*.

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} b\rho\, dV + \int_{\text{CS}} b\rho(V_r \cdot \vec{n})\, dA$$

$$B = E \qquad b = e \qquad b = e$$

$$\frac{dE_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \int_{\text{CS}} e\rho(\vec{V}_r \cdot \vec{n})\, dA$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{dE_{\text{sys}}}{dt}$$

$$e = u + \text{ke} + \text{pe} = u + V^2/2 + gz$$

$$\frac{dE_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \int_{\text{CS}} e\rho(\vec{V}_r \cdot \vec{n})A$$

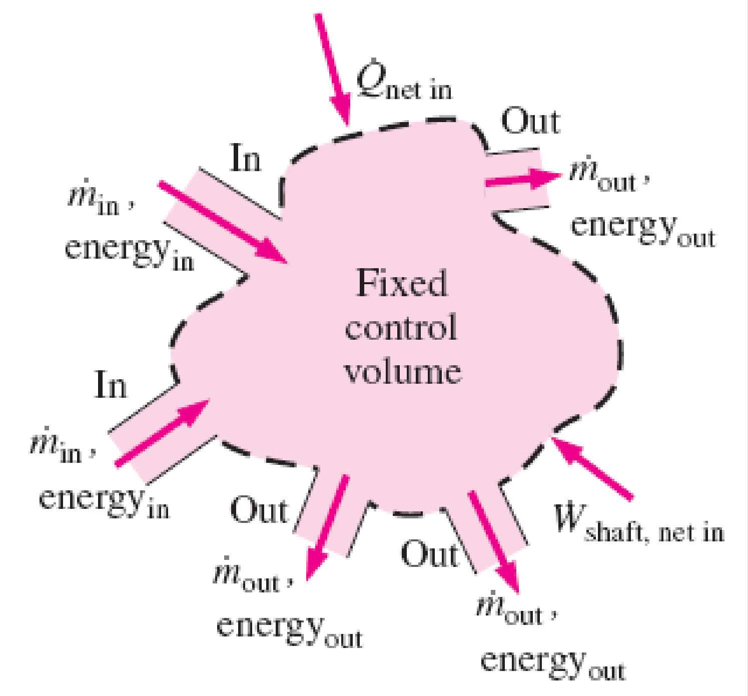
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \int_{\text{CS}} e\rho(\vec{V}_r \cdot \vec{n})\, dA$$

$$\begin{pmatrix}\text{The net rate of energy}\\ \text{transfer into a CV by}\\ \text{heat and work transfer}\end{pmatrix} = \begin{pmatrix}\text{The time rate of}\\ \text{change of the energy}\\ \text{content of the CV}\end{pmatrix} + \begin{pmatrix}\text{The net flow rate of}\\ \text{energy out of the control}\\ \text{surface by mass flow}\end{pmatrix}$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \int_{\text{CS}} \left(\frac{P}{\rho} + e\right)\rho(\vec{V}_r \cdot \vec{n})\, dA$$

Fixed CV: $$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \int_{\text{CS}} \left(\frac{P}{\rho} + e\right)\rho(\vec{V} \cdot \vec{n})\, dA$$

In a typical engineering problem, the control volume may contain many inlets and outlets; energy flows in at each inlet, and energy flows out at each outlet. Energy also enters the control volume through net heat transfer and net shaft work.



$$\dot{m} = \int_{A_c} \rho(\vec{V} \cdot \vec{n})\, dA_c$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, d\forall + \sum_{\text{out}} \dot{m}\left(\frac{P}{\rho} + e\right) - \sum_{\text{in}} \dot{m}\left(\frac{P}{\rho} + e\right)$$

$$e = u + V^2/2 + gz$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, d\forall + \sum_{\text{out}} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right)$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, d\forall + \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

$$h = u + P\nu = u + P/\rho$$

5–6 ■ ENERGY ANALYSIS OF STEADY FLOWS

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

The net rate of energy transfer to a control volume by heat transfer and work during steady flow is equal to the difference between the rates of outgoing and incoming energy flows by mass flow.

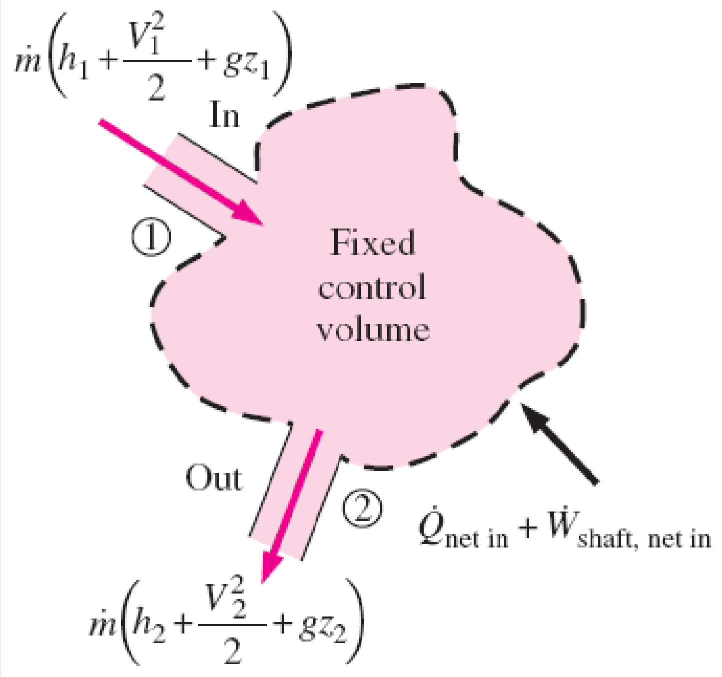
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

single-stream devices

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$h = u + P\text{v} = u + P/\rho$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}})$$



A control volume with only one inlet and one outlet and energy interactions.

Ideal flow (no mechanical energy loss):

$$q_{\text{net in}} = u_2 - u_1$$

Real flow (with mechanical energy loss):

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}}$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech, loss}}$$

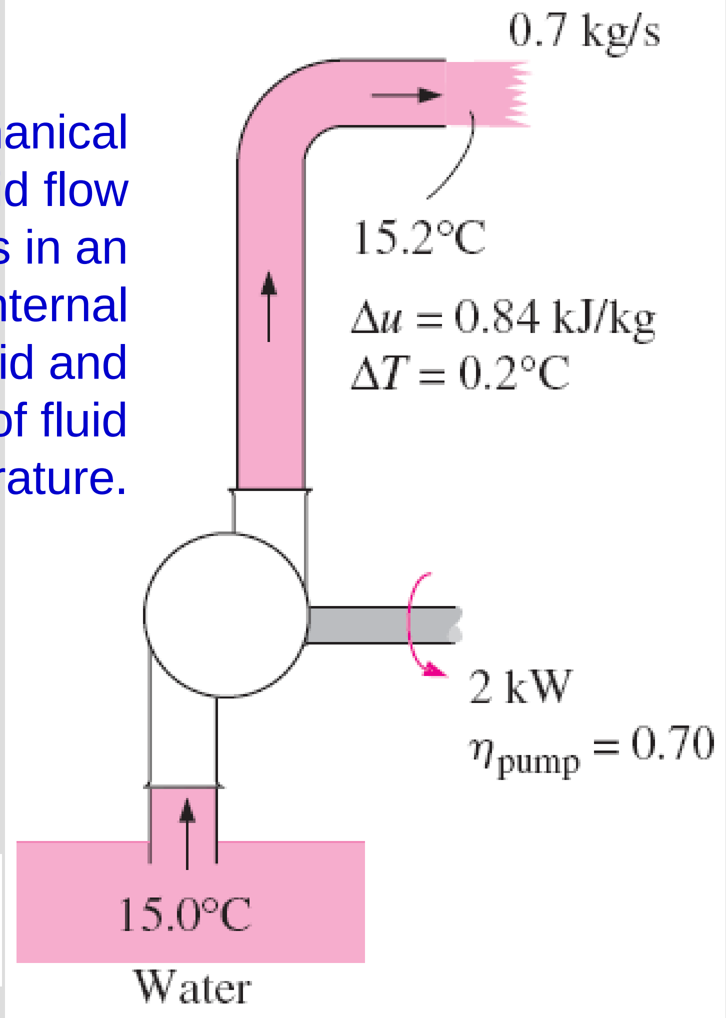
$$w_{\text{shaft, net in}} = w_{\text{pump}} - w_{\text{turbine}}$$

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$

The lost mechanical energy in a fluid flow system results in an increase in the internal energy of the fluid and thus in a rise of fluid temperature.



A typical power plant has numerous pipes, elbows, valves, pumps, and turbines, all of which have irreversible losses.

Energy equation in terms of *heads*

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L$$

where

- $h_{\text{pump, }u} = \dfrac{w_{\text{pump, }u}}{g} = \dfrac{\dot{W}_{\text{pump, }u}}{\dot{m}g} = \dfrac{\eta_{\text{pump}}\dot{W}_{\text{pump}}}{\dot{m}g}$ is the *useful head delivered to the fluid by the pump*. Because of irreversible losses in the pump, $h_{\text{pump, }u}$ is *less* than $\dot{W}_{\text{pump}}/\dot{m}g$ by the factor η_{pump}.
- $h_{\text{turbine, }e} = \dfrac{w_{\text{turbine, }e}}{g} = \dfrac{\dot{W}_{\text{turbine, }e}}{\dot{m}g} = \dfrac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$ is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, $h_{\text{turbine, }e}$ is greater than $\dot{W}_{\text{turbine}}/\dot{m}g$ by the factor η_{turbine}.
- $h_L = \dfrac{e_{\text{mech loss, piping}}}{g} = \dfrac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$ is the irreversible *head loss* between 1 and 2 due to all components of the piping system other than the pump or turbine.

$$\frac{P_1}{\rho_1 g}+\frac{V_1^2}{2g}+z_1+h_{\text{pump, }u}=\frac{P_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2+h_{\text{turbine, }e}+h_L \quad (5\text{-}74)$$

Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus $h_L = e_{\text{mech loss, piping}}/g \cong 0$. Also, $h_{\text{pump, }u} = h_{\text{turbine, }e} = 0$ when there are no mechanical work devices such as fans, pumps, or turbines. Then Eq. 5–74 reduces to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \qquad \text{or} \qquad \frac{P}{\rho g}+\frac{V^2}{2g}+z=\text{constant}$$

This is the **Bernoulli equation** derived earlier using Newton's second law of motion.

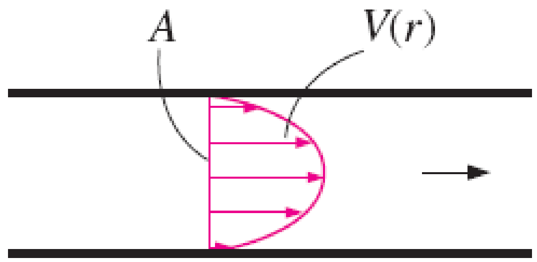
Thus, the Bernoulli equation can be thought of as a degenerate form of the energy equation.

Kinetic Energy Correction Factor, α

The kinetic energy of a fluid stream obtained from $V^2/2$ is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components.

This error can be corrected by replacing the kinetic energy terms $V^2/2$ in the energy equation by $\alpha V_{avg}^2/2$, where α is the **kinetic energy correction factor**.

The correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.



$$\dot{m} = \rho V_{avg} A, \qquad \rho = \text{constant}$$

$$\dot{KE}_{act} = \int ke\,\delta\dot{m} = \int_A \frac{1}{2}[V(r)]^2 [\rho V(r)\, dA]$$

$$= \frac{1}{2}\rho \int_A [V(r)]^3\, dA$$

$$\dot{KE}_{avg} = \frac{1}{2}\dot{m}V_{avg}^2 = \frac{1}{2}\rho A V_{avg}^3$$

$$\alpha = \frac{\dot{KE}_{act}}{\dot{KE}_{avg}} = \frac{1}{A}\int_A \left(\frac{V(r)}{V_{avg}}\right)^3 dA$$

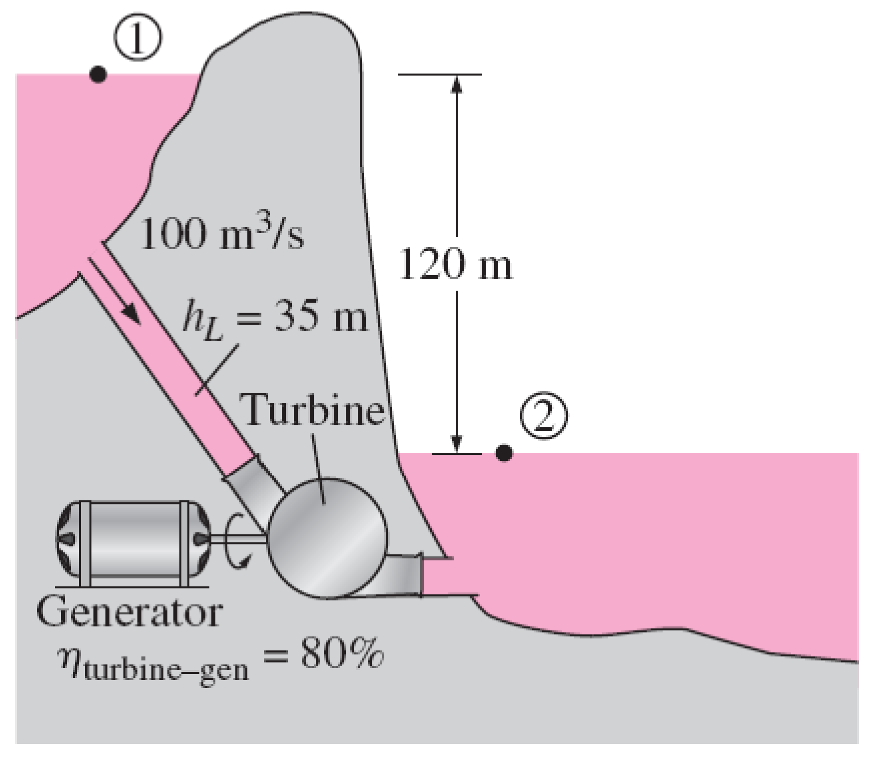
The determination of the *kinetic energy correction factor* using the actual velocity distribution $V(r)$ and the average velocity V_{avg} at a cross section.

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right) + \dot{W}_{pump} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right) + \dot{W}_{turbine} + \dot{E}_{mech,\ loss}$$

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{pump,\ u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{turbine,\ e} + h_L$$

EXAMPLE 5–13 Hydroelectric Power Generation from a Dam

In a hydroelectric power plant, 100 m³/s of water flows from an elevation of 120 m to a turbine, where electric power is generated (Fig. 5–55). The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output.



$$h_{\text{turbine}, e} = z_1 - h_L$$

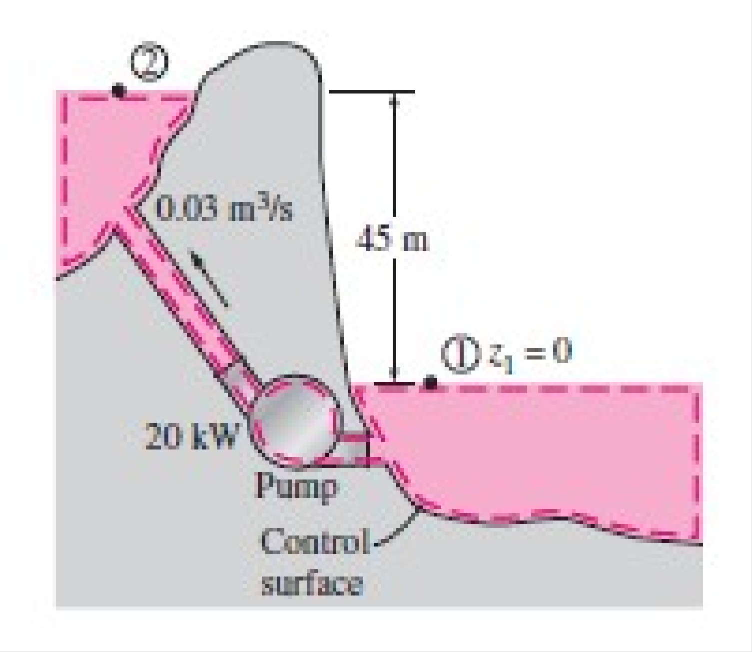
$$\dot{W}_{\text{electric}} = \eta_{\text{turbine–gen}} \dot{W}_{\text{turbine}, e}$$

$$\dot{W}_{\text{turbine}, e} = \dot{m} g h_{\text{turbine}, e}$$

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_{\text{pump}, u}}^{\,0} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}^{\,0} + h_{\text{turbine}, e} + h_L$$

EXAMPLE 5–15 Head and Power Loss During Water Pumping

Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to the water (Fig. 5–57). The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m^3/s, determine the irreversible head loss of the system and the lost mechanical power during this process.



Example: Pumping Water from a Lake to a Reservoir

Energy equation between 1 and 2

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump},\,u} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right)$$
$$+ \dot{W}_{\text{turbine},\,e} + \dot{E}_{\text{mech loss, piping}}$$

$$\dot{W}_{\text{pump},\,u} = \dot{m}gz_2 + \dot{E}_{\text{mech loss, piping}}$$

$$\dot{E}_{\text{mech loss, piping}} = \dot{m}gh_L$$

For the pump

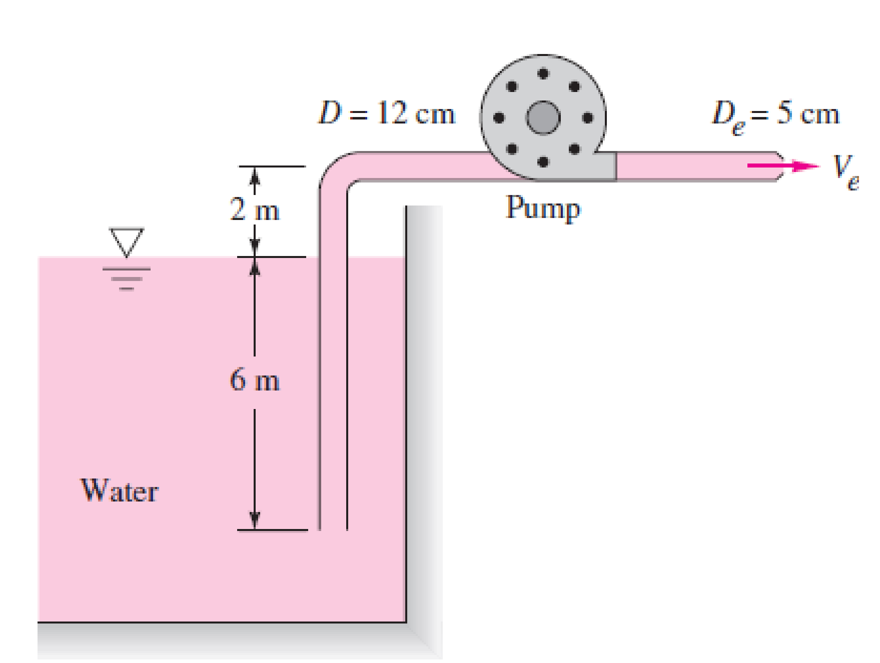
$$\Delta P = P_{\text{out}} - P_{\text{in}} = \frac{\dot{W}_{\text{pump},\,u}}{\dot{V}}$$

Example

Gasoline at 20°C is pumped through a smooth 12-cm-diameter pipe 10 km long, at a flow rate of 75 m^3/h (330 gal/min). The inlet is fed by a pump at an absolute pressure of 24 atm. The exit is at standard atmospheric pressure and is 150 m higher. Estimate the frictional head loss h_f, and compare it to the velocity head of the flow $V^2/(2g)$. (These numbers are quite realistic for liquid flow through long pipelines.)

Example

When the pump in Fig. P3.169 draws 220 m^3/h of water at 20°C from the reservoir, the total friction head loss is 5 m. The flow discharges through a nozzle to the atmosphere. Estimate the pump power in kW delivered to the water.



Example

5–76 A 7-hp (shaft) pump is used to raise water to a 15-m higher elevation. If the mechanical efficiency of the pump is 82 percent, determine the maximum volume flow rate of water.