

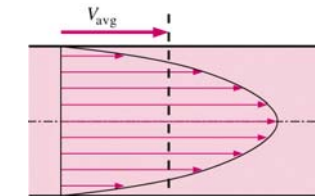
Chapter 8: Flow in Pipes

Objectives

1. Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
2. Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements
3. Understand the different velocity and flow rate measurement techniques and learn their advantages and disadvantages

Introduction

- Average velocity in a pipe
 - Recall - because of the no-slip condition, the velocity at the walls of a pipe or duct flow is zero
 - We are often interested only in V_{avg} , which we usually call just V (drop the subscript for convenience)
 - Keep in mind that the no-slip condition causes shear stress and friction along the pipe walls



Introduction

- For pipes of constant diameter and incompressible flow
 - V_{avg} stays the same down the pipe, even if the velocity profile changes
 - Why? Conservation of Mass

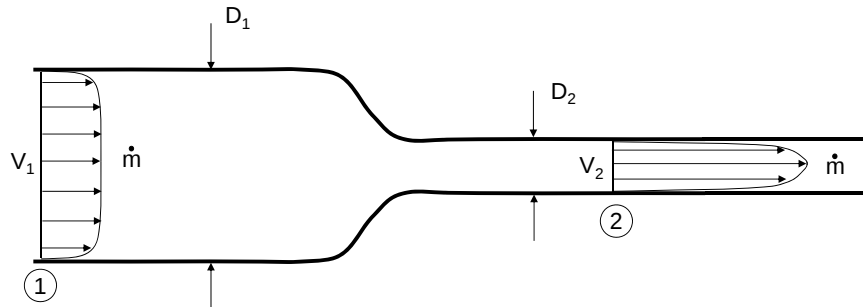


$$\dot{m} = \rho V_{avg} A = \text{constant}$$

same same same

Introduction

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$



Laminar and Turbulent Flows

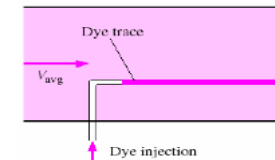
Laminar Flow

Can be steady or unsteady.

(Steady means that the flow field at any instant in time is the same as at any other instant in time.)

Can be one-, two-, or three-dimensional.

Has regular, *predictable* behavior



Analytical solutions are possible (see Chapter 9).

Occurs at *low* Reynolds numbers.

Turbulent Flow

Is always *unsteady*.

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.

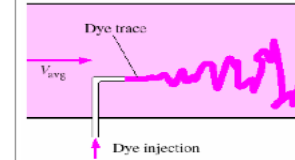
Note: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.

Is always *three-dimensional*.

Why? Again because of the random swirling eddies, which are in all directions.

Note: However, a turbulent flow can be 1-D or 2-D *in the mean*.

Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow).

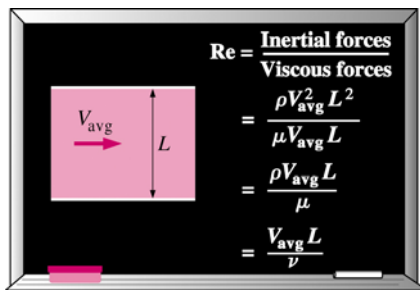


No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)

Occurs at *high* Reynolds numbers.

Laminar and Turbulent Flows

Definition of Reynolds number



- Critical Reynolds number (Re_{cr}) for flow in a round pipe

$Re < 2300 \Rightarrow$ laminar

$2300 \leq Re \leq 4000 \Rightarrow$ transitional

$Re > 4000 \Rightarrow$ turbulent

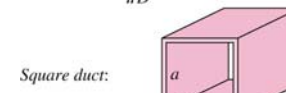
- Note that these values are approximate.
- For a given application, Re_{cr} depends upon
 - Pipe roughness
 - Vibrations
 - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

Laminar and Turbulent Flows



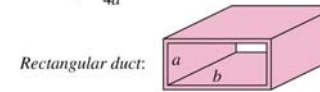
Circular tube:

$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$



Square duct:

$$D_h = \frac{4a^2}{4a} = a$$



Rectangular duct:

$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

- For non-round pipes, define the hydraulic diameter

$$D_h = 4A_c/P$$

A_c = cross-section area

P = wetted perimeter

- Example: open channel

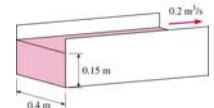
$$A_c = 0.15 \times 0.4 = 0.06 \text{ m}^2$$

$$P = 0.15 + 0.15 + 0.4 = 0.7 \text{ m}$$

Don't count free surface, since it does not contribute to friction along pipe walls!

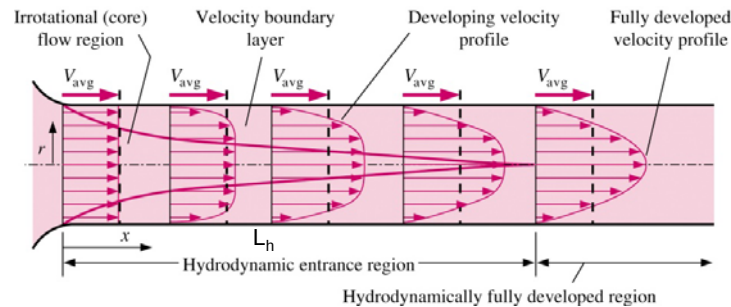
$$D_h = 4A_c/P = 4 \times 0.06/0.7 = 0.34 \text{ m}$$

What does it mean? This channel flow is equivalent to a round pipe of diameter 0.34 m (approximately).



The Entrance Region

- Consider a round pipe of diameter D . The flow can be laminar or turbulent. In either case, the profile develops downstream over several diameters called the *entry length* L_h . L_h/D is a function of Re .



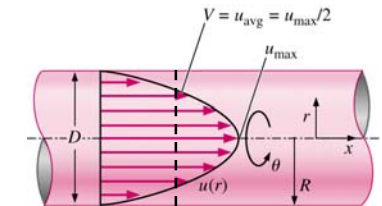
Fully Developed Pipe Flow

- Comparison of laminar and turbulent flow

There are some major differences between laminar and turbulent fully developed pipe flows

Laminar

- Can solve exactly (Chapter 9)
- Flow is steady
- Velocity profile is parabolic
- Pipe roughness not important

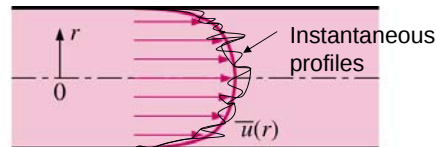


It turns out that $V_{avg} = 1/2 U_{max}$ and $u(r) = 2V_{avg}(1 - r^2/R^2)$

Fully Developed Pipe Flow

Turbulent

- Cannot solve exactly (too complex)
- Flow is unsteady (3D swirling eddies), but it is steady in the mean
- Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall)
- Pipe roughness is very important



- V_{avg} 85% of U_{max} (depends on Re a bit)
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape. See text
 - Logarithmic law (Eq. 8-46)
 - Power law (Eq. 8-49)

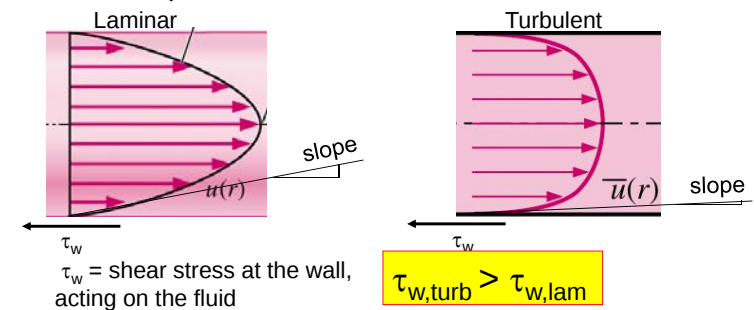
Fully Developed Pipe Flow Wall-shear stress

- Recall, for simple shear flows $u=u(y)$, we had

$$\tau = \mu \, du/dy$$

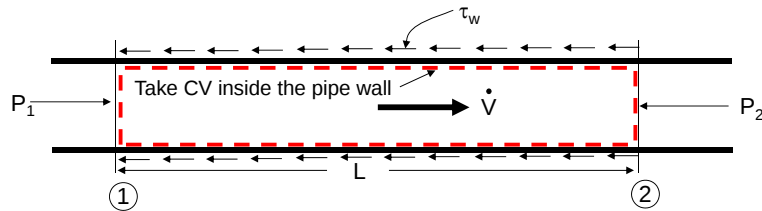
- In fully developed pipe flow, it turns out that

$$\tau = \mu \, du/dr$$



Fully Developed Pipe Flow Pressure drop

- There is a direct connection between the pressure drop in a pipe and the shear stress at the wall
- Consider a horizontal pipe, fully developed, and incompressible flow



- Let's apply conservation of mass, momentum, and energy to this CV (good review problem!)

Fully Developed Pipe Flow Pressure drop

- Conservation of Mass

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\rho \dot{V}_1 = \rho \dot{V}_2 \rightarrow \dot{V} = \text{const}$$

$$V_1 \frac{\pi D^2}{4} = V_2 \frac{\pi D^2}{4} \rightarrow V_1 = V_2$$

- Conservation of x-momentum

$$\sum F_x = \sum F_{x,grav} + \sum F_{x,press} + \sum F_{x,visc} + \sum F_{x,other} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$$

$$P_1 \frac{\pi D^2}{4} - P_2 \frac{\pi D^2}{4} - \tau_w \pi D L = \underbrace{\beta_2 \dot{m} V_2 - \beta_1 \dot{m} V_1}_{\text{Terms cancel since } \beta_1 = \beta_2 \text{ and } V_1 = V_2}$$

Fully Developed Pipe Flow Pressure drop

- Thus, x-momentum reduces to

$$(P_1 - P_2) \frac{\pi D^2}{4} = \tau_w \pi D L \quad \text{or} \quad P_1 - P_2 = 4 \tau_w \frac{L}{D}$$

- Energy equation (in head form)

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L$$

cancel (horizontal pipe)

Velocity terms cancel again because $V_1 = V_2$, and $\alpha_1 = \alpha_2$ (shape not changing)

$$P_1 - P_2 = \rho g h_L$$

h_L = irreversible head loss & it is felt as a pressure drop in the pipe

Fully Developed Pipe Flow Friction Factor

- From momentum CV analysis

$$P_1 - P_2 = 4 \tau_w \frac{L}{D}$$

- From energy CV analysis

$$P_1 - P_2 = \rho g h_L$$

- Equating the two gives

$$4 \tau_w \frac{L}{D} = \rho g h_L \quad h_L = \frac{4 \tau_w}{\rho g} \frac{L}{D}$$

- To predict head loss, we need to be able to calculate τ_w . How?
 - Laminar flow: solve exactly
 - Turbulent flow: rely on empirical data (experiments)
 - In either case, we can benefit from dimensional analysis!

Fully Developed Pipe Flow Friction Factor

- $\tau_w = \text{func}(\rho, V, \mu, D, \varepsilon)$ ε = average roughness of the inside wall of the pipe
- Π -analysis gives

$\Pi_1 = f$
 $\Pi_2 = Re$
 $\Pi_3 = \frac{\varepsilon}{D}$
 $\Pi_1 = \text{func}(\Pi_2, \Pi_3)$

$f = \frac{8\tau_w}{\rho V^2}$

$Re = \frac{\rho V D}{\mu}$

$\varepsilon/D = \text{roughness factor}$

$f = \text{func}(Re, \varepsilon/D)$

Fully Developed Pipe Flow Friction Factor

- Now go back to equation for h_L and substitute f for τ_w

$$h_L = \frac{4\tau_w}{\rho g} \frac{L}{D} \quad f = \frac{8\tau_w}{\rho V^2} \rightarrow \tau_w = f \rho V^2 / 8$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

- Our problem is now reduced to solving for Darcy friction factor f
 - Recall $f = \text{func}(Re, \varepsilon/D)$ But for laminar flow, roughness does not affect the flow unless it is huge
 - Therefore
 - Laminar flow: $f = 64/Re$ (exact)
 - Turbulent flow: Use charts or empirical equations (Moody Chart, a famous plot of f vs. Re and ε/D)

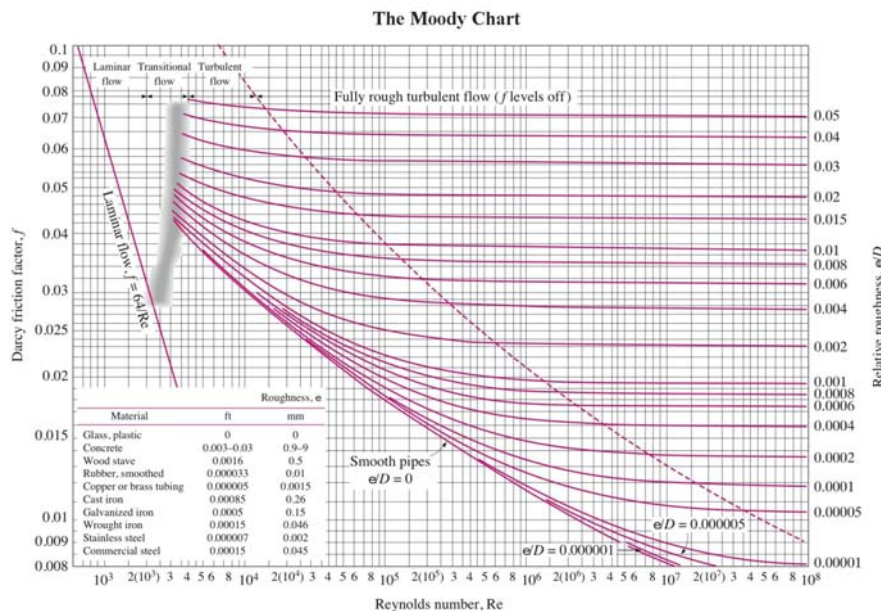
Fully Developed Pipe Flow Friction Factor

- Moody chart was developed for circular pipes, but can be used for non-circular pipes using hydraulic diameter
- Colebrook equation is a curve-fit of the data which is convenient for computations (e.g., using EES)

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

Implicit equation for f which can be solved using the root-finding algorithm in EES

- Both Moody chart and Colebrook equation are accurate to $\pm 15\%$ due to roughness size, experimental error, curve fitting of data, etc.



Types of Fluid Flow Problems

- In design and analysis of piping systems, 3 problem types are encountered
 1. Determine Δp (or h_L) given L , D , V (or flow rate)
Can be solved directly using Moody chart and Colebrook equation
 2. Determine V , given L , D , Δp
 3. Determine D , given L , Δp , V (or flow rate)
- Types 2 and 3 are common engineering design problems, i.e., selection of pipe diameters to minimize construction and pumping costs
- However, iterative approach required since both V and D are in the Reynolds number.

Types of Fluid Flow Problems

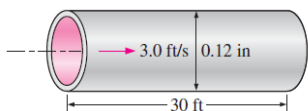
- Explicit relations have been developed which eliminate iteration. They are useful for quick, direct calculation, but introduce an additional 2% error

$$h_L = 1.07 \frac{\dot{V}^2 L}{g D^5} \left\{ \ln \left[\frac{\epsilon}{3.7 D} + 4.62 \left(\frac{\nu D}{\dot{V}} \right)^{0.9} \right] \right\}^{-2} \quad \begin{matrix} 10^{-6} < \epsilon/D < 10^{-2} \\ 3000 < Re < 3 \times 10^8 \end{matrix}$$

$$\dot{V} = -0.965 \left(\frac{g D^5 h_L}{L} \right)^{0.5} \ln \left[\frac{\epsilon}{3.7 D} + \left(\frac{3.17 \nu^2 L}{g D^3 h_L} \right)^{0.5} \right] \quad Re > 2000$$

$$D = 0.66 \left[\epsilon^{1.25} \left(\frac{L \dot{V}^2}{g h_L} \right)^{4.75} + \nu \dot{V}^{9.4} \left(\frac{L}{g h_L} \right)^{5.2} \right]^{0.04} \quad \begin{matrix} 10^{-6} < \epsilon/D < 10^{-2} \\ 5000 < Re < 3 \times 10^8 \end{matrix}$$

EXAMPLE 8–2 Pressure Drop and Head Loss in a Pipe



Water at 40°F ($\rho = 62.42 \text{ lbm/ft}^3$ and $\mu = 1.038 \times 10^{-3} \text{ lbm/ft} \cdot \text{s}$) is flowing through a 0.12-in-(0.010 ft) diameter 30-ft-long horizontal pipe steadily at an average velocity of 3.0 ft/s.

Determine :

- (a) the head loss
- (b) the pressure drop
- (c) the pumping power requirement to overcome this pressure drop

Assumption: The entrance effects are negligible, and thus the flow is fully developed.

Solution :

$$Re = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})(0.01 \text{ ft})}{1.038 \times 10^{-3} \text{ lbm/ft} \cdot \text{s}} = 1803 \quad \text{The flow is laminar.}$$

$$f = \frac{64}{Re} = \frac{64}{1803} = 0.0355$$

$$h_L = f \frac{L}{D} \frac{V_{\text{avg}}^2}{2g} = 0.0355 \frac{30 \text{ ft}}{0.01 \text{ ft}} \frac{(3 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 14.9 \text{ ft}$$

EXAMPLE 8–2 Pressure Drop and Head Loss in a Pipe

$$\begin{aligned} \Delta P = \Delta P_L &= f \frac{L}{D} \frac{\rho V_{\text{avg}}^2}{2} = 0.0355 \frac{30 \text{ ft}}{0.01 \text{ ft}} \frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})^2}{2} \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \\ &= 929 \text{ lbf/ft}^2 = 6.45 \text{ psi} \end{aligned}$$

➤ The volume flow rate and the pumping power requirements are

$$\dot{V} = V_{\text{avg}} A_c = V_{\text{avg}} (\pi D^2/4) = (3 \text{ ft/s}) [\pi (0.01 \text{ ft})^2/4] = 0.000236 \text{ ft}^3/\text{s}$$

$$\dot{W}_{\text{pump}} = \dot{V} \Delta P = (0.000236 \text{ ft}^3/\text{s})(929 \text{ lbf/ft}^2) \left(\frac{1 \text{ W}}{0.737 \text{ lbf} \cdot \text{ft/s}} \right) = 0.30 \text{ W}$$

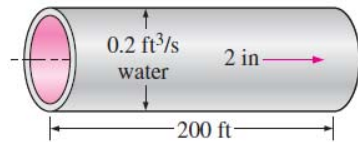
Therefore, power input in the amount of 0.30 W is needed to overcome the frictional losses in the flow due to viscosity.

EXAMPLE 8–3 Determining the Head Loss in a Water Pipe

Water at 60°F ($\rho = 62.36 \text{ lbm/ft}^3$ and $\mu = 7.536 \times 10^{-4} \text{ lbm/ft} \cdot \text{s}$) is flowing steadily in a 2-in-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft³/s.

Determine:

- the pressure drop
- the head loss
- the required pumping power input for flow over a 200-ft-long section of the pipe.



Assumption: The entrance effects are negligible, and thus the flow is fully developed.

EXAMPLE 8–3 Determining the Head Loss in a Water Pipe

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2 \text{ ft}^3/\text{s}}{\pi(2/12 \text{ ft})^2/4} = 9.17 \text{ ft/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.36 \text{ lbm/ft}^3)(9.17 \text{ ft/s})(2/12 \text{ ft})}{7.536 \times 10^{-4} \text{ lbm/ft} \cdot \text{s}} = 126,400$$

$$\varepsilon/D = \frac{0.000007 \text{ ft}}{2/12 \text{ ft}} = 0.000042$$

Colebrook equation: $\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{0.000042}{3.7} + \frac{2.51}{126,400 \sqrt{f}} \right)$

$$f = 0.0174$$

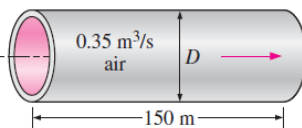
$$\Delta P = \Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2} = 0.0174 \frac{200 \text{ ft}}{2/12 \text{ ft}} \frac{(62.36 \text{ lbm/ft}^3)(9.17 \text{ ft/s})^2}{2} \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right)$$

$$= 1700 \text{ lbf/ft}^2 = 11.8 \text{ psi}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V^2}{2g} = 0.0174 \frac{200 \text{ ft}}{2/12 \text{ ft}} \frac{(9.17 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 27.3 \text{ ft}$$

$$\dot{W}_{\text{pump}} = \dot{V} \Delta P = (0.2 \text{ ft}^3/\text{s})(1700 \text{ lbf/ft}^2) \left(\frac{1 \text{ W}}{0.737 \text{ lbf} \cdot \text{ft/s}} \right) = 461 \text{ W}$$

EXAMPLE 8–4 Determining the Diameter of an Air Duct



Heated air at 1 atm and 35°C is to be transported in a circular plastic duct. If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.

Assumption:

- The entrance effects are negligible, and thus the flow is fully developed.
- Air is an ideal gas.
- The duct is smooth since it is made of plastic.
- The flow is turbulent (to be verified)

Properties The density, dynamic viscosity, and kinematic viscosity of air at 35°C are $\rho = 1.145 \text{ kg/m}^3$, $\mu = 1.895 \times 10^{-5} \text{ kg/m} \cdot \text{s}$, and $\nu = 1.655 \times 10^{-5} \text{ m}^2/\text{s}$.

EXAMPLE 8–4 Determining the Diameter of an Air Duct

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.35 \text{ m}^3/\text{s}}{\pi D^2/4}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{VD}{1.655 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right) = -2.0 \log \left(\frac{2.51}{\text{Re} \sqrt{f}} \right)$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g} \rightarrow 20 = f \frac{150 \text{ m}}{D} \frac{V^2}{2(9.81 \text{ m/s}^2)}$$

Therefore, this is a set of four equations in four unknowns, and solving them with an equation solver such as EES gives

$$D = 0.267 \text{ m}, \quad f = 0.0180, \quad V = 6.24 \text{ m/s}, \quad \text{and} \quad \text{Re} = 100,800$$

Therefore, the diameter of the duct should be more than 26.7 cm if the head loss is not to exceed 20 m. Note that $\text{Re} > 4000$, and thus the turbulent flow assumption is verified.

EXAMPLE 8-4 Determining the Diameter of an Air Duct

Alternative solution strategy for the problem : iterative approach

- Set f equal to an initial value. (for example 0.02)
- Calculate Re from Colebrook eq.

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right) = -2.0 \log \left(\frac{2.51}{Re \sqrt{f}} \right)$$

- Calculate V from eqs (1) and (2)

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad (1)$$

$$Re = \frac{VD}{\nu} \quad (2)$$

- Calculate D by below relation. And iterate operations until convergence.

$$h_L = f \frac{L}{D} \frac{V^2}{2g} \rightarrow 20 = f \frac{150 \text{ m}}{D} \frac{V^2}{2(9.81 \text{ m/s}^2)}$$

Minor Losses

- Piping systems include fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions.
- These components interrupt the smooth flow of fluid and cause additional losses because of flow separation and mixing
- We introduce a relation for the minor losses associated with these components

$$h_L = K_L \frac{V^2}{2g}$$

- K_L is the loss coefficient.
- Is different for each component.
- Is assumed to be independent of Re .
- Typically provided by manufacturer or generic table (e.g., Table 8-4 in text).

Minor Losses

- Total head loss in a system is comprised of major losses (in the pipe sections) and the minor losses (in the components)

$$h_L = h_{L,major} + h_{L,minor}$$

$$h_L = \underbrace{\sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g}}_{i \text{ pipe sections}} + \underbrace{\sum_j K_{L,j} \frac{V_j^2}{2g}}_{j \text{ components}}$$

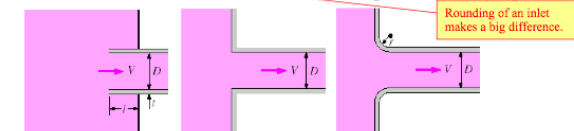
- If the piping system has constant diameter

$$h_L = \left(f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

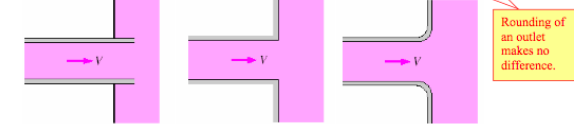
Minor Losses

Here are some sample loss coefficients for various minor loss components. More values are listed in Table 8-4, page 350 of the Çengel-Cimbala textbook:

Pipe Inlet
 Reentrant: $K_L = 0.80$ ($t \ll D$ and $l \approx 0.1D$)
 Sharp-edged: $K_L = 0.50$
 Well-rounded ($r/D > 0.2$): $K_L = 0.03$
 Slightly rounded ($r/D = 0.1$): $K_L = 0.12$ (see Fig. 8-36)

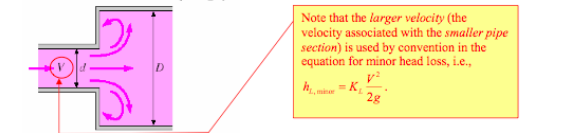


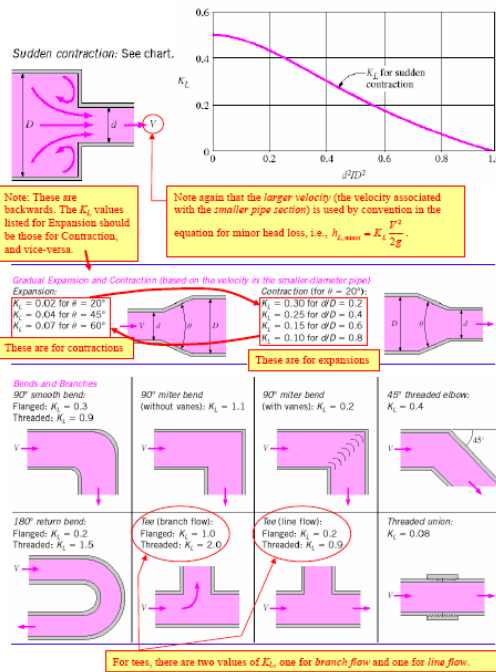
Pipe Exit
 Reentrant: $K_L = \alpha$
 Sharp-edged: $K_L = \alpha$
 Rounded: $K_L = \alpha$



Sudden Expansion and Contraction (based on the velocity in the smaller diameter pipe)

Sudden expansion: $K_L = \left(1 - \frac{d^2}{D^2} \right)^2$



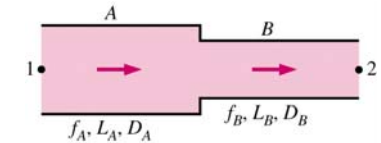


Piping Networks and Pump Selection

Two general types of networks

Pipes in series

- Volume flow rate is constant
- Head loss is the summation of parts

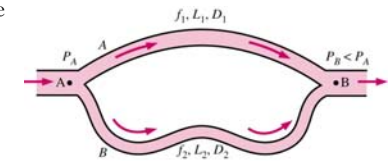


$$\dot{V}_A = \dot{V}_B$$

$$h_{L, 1-2} = h_{L, A} + h_{L, B}$$

Pipes in parallel

- Volume flow rate is the sum of the components
- Pressure loss across all branches is the same



$$h_{L, 1} = h_{L, 2}$$

$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

Piping Networks and Pump Selection

- For parallel pipes, perform CV analysis between points A and B

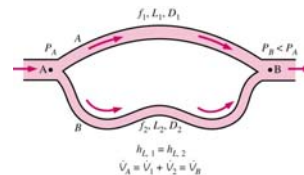
$$V_A = V_B$$

$$\frac{P_A}{\rho g} + \alpha_1 \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \alpha_2 \frac{V_B^2}{2g} + z_B + h_L$$

$$h_L = \frac{\Delta P}{\rho g}$$

- Since Δp is the same for all branches, head loss in all branches is the same

$$h_{L, 1} = h_{L, 2} \longrightarrow f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$



Piping Networks and Pump Selection

- Head loss relationship between branches allows the following ratios to be developed

Flow rate ratio

$$\frac{V_1}{V_2} = \left(\frac{f_2 L_2 D_1}{f_1 L_1 D_2} \right)^{\frac{1}{2}}$$

$$\frac{\dot{V}_1}{\dot{V}_2} = \frac{D_1^2}{D_2^2} \left(\frac{f_2 L_2 D_1}{f_1 L_1 D_2} \right)^{\frac{1}{2}}$$

- Real pipe systems result in a system of non-linear equations. Very easy to solve with EES!
- Note: the analogy with electrical circuits should be obvious
 - Flow flow rate (V_A) : current (I)
 - Pressure gradient (Δp) : electrical potential (V)
 - Head loss (h_L): resistance (R), however h_L is very nonlinear

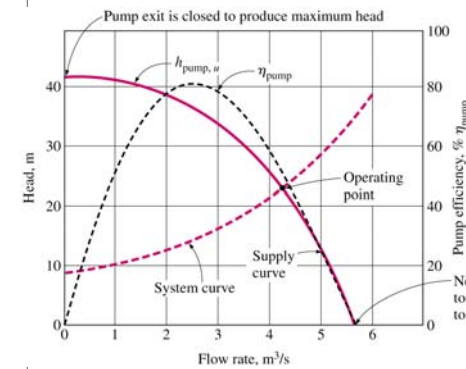
Piping Networks and Pump Selection

- When a piping system involves pumps and/or turbines, pump and turbine head must be included in the energy equation

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$

- The useful head of the pump ($h_{\text{pump},u}$) or the head extracted by the turbine ($h_{\text{turbine},e}$), are functions of volume flow rate, i.e., they are not constants.
- Operating point of system is where the system is in balance, e.g., where pump head is equal to the head losses.

Pump and systems curves



- Supply curve for $h_{\text{pump},u}$: determine experimentally by manufacturer. When using EES, it is easy to build in functional relationship for $h_{\text{pump},u}$.
- System curve determined from analysis of fluid dynamics equations
- Operating point is the intersection of supply and demand curves
- If peak efficiency is far from operating point, pump is wrong for that application.