5–55 A pressurized tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level is 2.5 m above the outlet. The tank air pressure above the water level is 250 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank. *Answer: 0.147 m³/s*

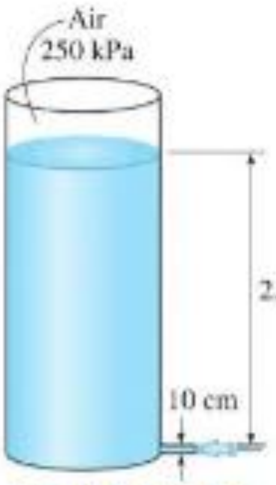


FIGURE P5–55

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{V_2^2}{2g}=\frac{P_1-P_2}{\rho g}+z_1$$

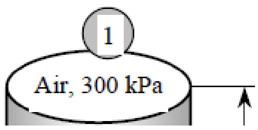
Solving for V_2 and substituting, the discharge velocity is determined to

$$V_2=\sqrt{\frac{2(P_1-P_2)}{\rho}+2gz_1}=\sqrt{\frac{2(300-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)+2(9.81\text{ m/s}^2)(3\text{ m})}$$

$$=21.4\text{ m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V}=A_{\text{orifice}}V_2=\frac{\pi D^2}{4}V_2=\frac{\pi(0.10\text{ m})^2}{4}(21.4\text{ m/s})=\mathbf{0.168\ m^3/s}$$



5–58 The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, and the air pressure above the water surface is 2 atm gage. The system is at sea level. Determine the maximum height to which the water stream could rise. *Answer:* 40.7 m

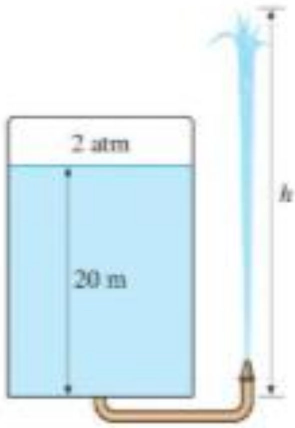


FIGURE P5–58

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}+z_1=\frac{P_{atm}}{\rho g}+z_2 \quad\rightarrow\quad z_2=\frac{P_1-P_{atm}}{\rho g}+z_1=\frac{P_{1,\text{gage}}}{\rho g}+z_1$$

Substituting,

$$z_2=\frac{2\text{ atm}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{101{,}325\text{ N/m}^2}{1\text{ atm}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)+20=\mathbf{40.7\ m}$$

5–84 Water enters a hydraulic turbine through a 30-cm-diameter pipe at a rate of 0.6 m³/s and exits through a 25-cm-diameter pipe. The pressure drop in the turbine is measured by a mercury manometer to be 1.2 m. For a combined turbine– generator efficiency of 83 percent, determine the net electric power output. Disregard the effect of the kinetic energy correction factors.

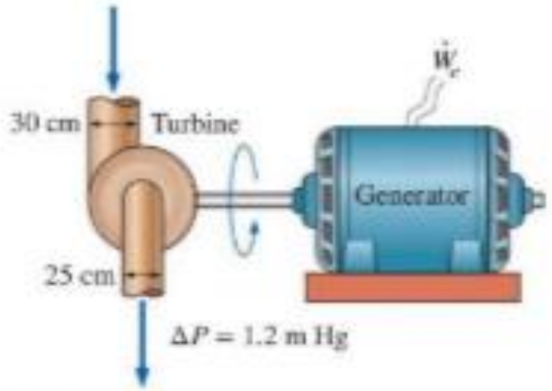
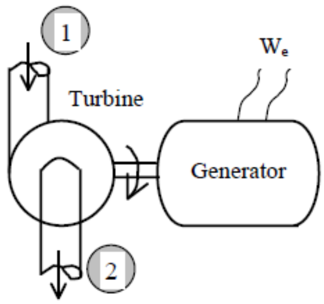


FIGURE P5–84

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{turbine, e}}=\frac{P_1-P_2}{\rho_{\text{water}}g}+\frac{\alpha(V_1^2-V_2^2)}{2g} \qquad (1)$$

where



$$V_1=\frac{\dot V}{A_1}=\frac{\dot V}{\pi D_1^2/4}\frac{0.6\ \text{m}^3/\text{s}}{\pi(0.30\ \text{m})^2/4}=8.49\ \text{m/s}$$

$$V_2=\frac{\dot V}{A_2}=\frac{\dot V}{\pi D_2^2/4}=\frac{0.6\ \text{m}^3/\text{s}}{\pi(0.25\ \text{m})^2/4}=12.2\ \text{m/s}$$

The pressure drop corresponding to a differential height of 1.2 m in the mercury manometer is

$$\begin{aligned}P_1-P_2&=(\rho_{\text{Hg}}-\rho_{\text{water}})gh\\&=[(13{,}560-1000)\ \text{kg/m}^3](9.81\ \text{m/s}^2)(1.2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\\&=148\ \text{kN/m}^2=148\ \text{kPa}\end{aligned}$$

Substituting into Eq. (1), the turbine head is determined to be

$$h_{\text{turbine, e}}=\frac{148\ \text{kN/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right)+(1.0)\frac{(8.49\ \text{m/s})^2-(12.2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=15.1-3.9=11.2\ \text{m}$$

Then the net electric power output of this hydroelectric turbine becomes

$$\begin{aligned}\dot W_{\text{turbine}}&=\eta_{\text{turbine-gen}}\dot m g h_{\text{turbine, e}}=\eta_{\text{turbine-gen}}\rho\dot V g h_{\text{turbine, e}}\\&=0.83(1000\ \text{kg/m}^3)(0.6\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(11.2\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)=\mathbf{55\ kW}\end{aligned}$$

5–109 An oil pump is drawing 18 kW of electric power while pumping oil with $\rho = 860$ kg/m^3 at a rate of 0.1 m^3/s. The inlet and outlet diameters of the pipe are 8 cm and 12 cm, respectively. If the pressure rise of oil in the pump is measured to be 250 kPa and the motor efficiency is 95 percent, determine the mechanical efficiency of the pump. Take the kinetic energy correction factor to be 1.05.

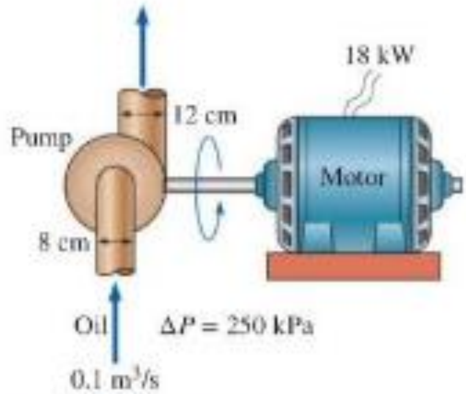
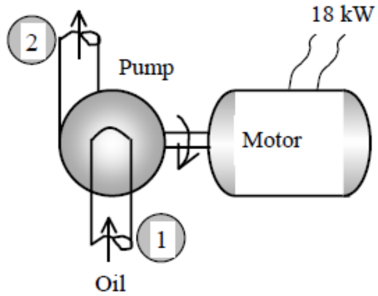


FIGURE P5–109

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=\frac{P_2-P_1}{\rho g}+\frac{\alpha(V_2^2-V_1^2)}{2g}$$

where



$$V_1=\frac{\dot V}{A_1}=\frac{\dot V}{\pi D_1^2/4}=\frac{0.1\,\text{m}^3/\text{s}}{\pi(0.08\,\text{m})^2/4}=19.9\text{ m/s}$$

$$V_2=\frac{\dot V}{A_2}=\frac{\dot V}{\pi D_2^2/4}=\frac{0.1\,\text{m}^3/\text{s}}{\pi(0.12\,\text{m})^2/4}=8.84\text{ m/s}$$

Substituting, the useful pump head and the corresponding useful pumping power are determined to be

$$h_{\text{pump, u}}=\frac{400{,}000\text{ N/m}^2}{(860\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)+\frac{1.05[(8.84\text{ m/s})^2-(19.9\text{ m/s})^2]}{2(9.81\text{ m/s}^2)}=47.4-17.0=30.4\text{ m}$$

$$\dot W_{\text{pump,u}}=\rho\dot V g h_{\text{pump, u}}=(860\text{ kg/m}^3)(0.1\text{ m}^3/\text{s})(9.81\text{ m/s}^2)(30.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right)=25.6\text{ kW}$$

Then the shaft pumping power and the mechanical efficiency of the pump become

$$\dot W_{\text{pump,shaft}}=\eta_{\text{motor}}\dot W_{\text{electric}}=(0.90)(35\text{ kW})=31.5\text{ kW}$$

$$\eta_{\text{pump}}=\frac{\dot W_{\text{pump, u}}}{\dot W_{\text{pump, shaft}}}=\frac{25.6\text{ kW}}{31.5\text{ kW}}=0.813=81.3\%$$

5–81 Water flows at a rate of 20 L/s through a horizontal pipe whose diameter is constant at 3 cm. The pressure drop across a valve in the pipe is measured to be 2 kPa, as shown in Fig. P5–81. Determine the irreversible head loss of the valve, and the useful pumping power needed to overcome the resulting pressure drop. *Answers: 0.204 m, 40 W*

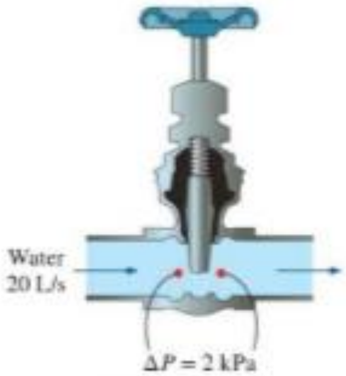
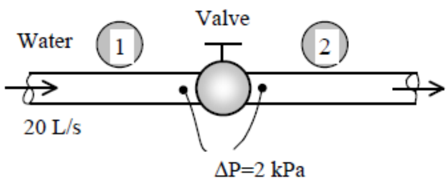


FIGURE P5–81

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_L=\frac{P_1-P_2}{\rho g}$$

Substituting,

$$h_L=\frac{2\text{ kN/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)=\mathbf{0.204\ m}$$



The useful pumping power needed to overcome this head loss is

$$\dot{W}_{\text{pump, u}}=\dot{m}gh_L=\rho\dot{V}gh_L$$

$$=(1000\text{ kg/m}^3)(0.020\text{ m}^3\text{/s})(9.81\text{ m/s}^2)(0.204\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right)=\mathbf{40\ W}$$

6–20 A 90° elbow in a horizontal pipe is used to direct water flow upward at a rate of 40 kg/s. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere, and thus the pressure at the exit is the local atmospheric pressure. The elevation difference between the centers of the exit and the inlet of the elbow is 50 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (*a*) the gage pressure at the center of the inlet of the elbow and (*b*) the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03 at both the inlet and the outlet.

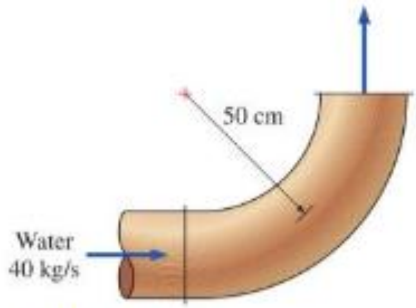


FIGURE P6–20

Analysis (*a*) We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30\text{ kg/s}$. Noting that $\dot{m} = \rho A V$, the mean inlet and outlet velocities of water are

$$V_1 = V_2 = V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\text{ kg/s}}{(1000\text{ kg/m}^3)[\pi(0.1\text{ m})^2/4]} = 3.18\text{ m/s}$$

Noting that $V_1 = V_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

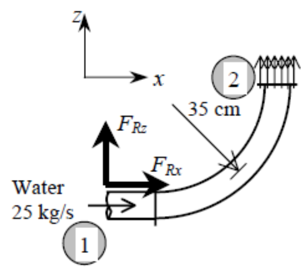
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g(z_2 - z_1) \rightarrow\; P_{1,\text{gage}} = \rho g(z_2 - z_1)$$

Substituting,

$$P_{1,\text{gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.35\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 3.434\text{ kN/m}^2 = \mathbf{3.434\text{ kPa}}$$

(*b*) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *y* axes become

$$F_{Rx} + P_{1,\text{gage}}A_1 = 0 - \beta\dot{m}(+V_1) = -\beta\dot{m}V$$
$$F_{Rz} = \beta\dot{m}(+V_2) = \beta\dot{m}V$$



Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = -\beta\dot{m}V - P_{1,\text{gage}}A_1$$
$$= -1.03(25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) - (3434\text{ N/m}^2)[\pi(0.1\text{ m})^2/4]$$
$$= -109\text{ N}$$
$$F_{Ry} = \beta\dot{m}V = 1.03(25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 81.9\text{ N}$$

and $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-109)^2 + 81.9^2} = \mathbf{136\text{ N}}$, $\quad \theta = \tan^{-1}\dfrac{F_{Ry}}{F_{Rx}} = \tan^{-1}\dfrac{81.9}{-109} = -37° = \mathbf{143°}$

6–24 A reducing elbow in a horizontal pipe is used to deflect water flow by an angle $\theta = 45°$ from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 150 cm² at the inlet and 25 cm² at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm. The mass of the elbow and the water in it is 50 kg. Determine the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03 at both the inlet and outlet.

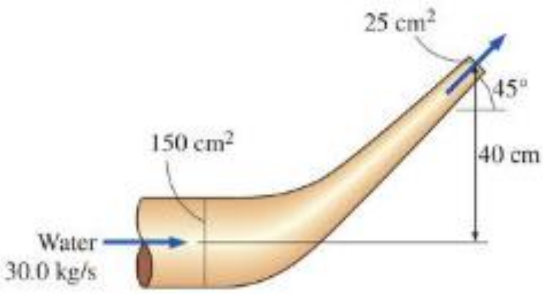


FIGURE P6–24

$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N} = 0.4905\text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30\text{ kg/s}$. Noting that $\dot{m} = \rho A V$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0150\text{ m}^2)} = 2.0\text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0025\text{ m}^2)} = 12\text{ m/s}$$

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

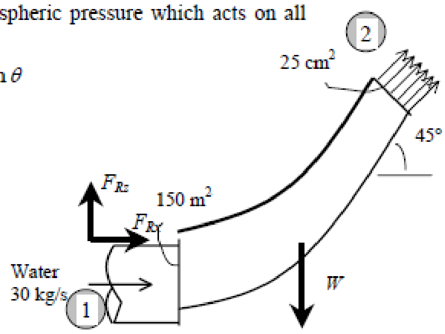
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right) \;\rightarrow\; P_{1,\text{gage}} = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$$

Substituting,

$$P_{1,\text{gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2 - (2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 73.9\text{ kN/m}^2 = 73.9\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$. We let the x- and z- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and z axes become

$$F_{Rx} + P_{1,\text{gage}}A_1 = \beta\dot{m}V_2\cos\theta - \beta\dot{m}V_1 \quad\text{and}\quad F_{Rz} - W = \beta\dot{m}V_2\sin\theta$$



Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta\dot{m}(V_2\cos\theta - V_1) - P_{1,\text{gage}}A_1$$

$$= 1.03(30\text{ kg/s})[(12\cos 45° - 2)\text{ m/s}]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$-(73.9\text{ kN/m}^2)(0.0150\text{ m}^2)$$

$$= -0.908\text{ kN}$$

$$F_{Rz} = \beta\dot{m}V_2\sin\theta + W = 1.03(30\text{ kg/s})(12\sin 45°\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) + 0.4905\text{ kN} = 0.753\text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-0.908)^2 + (0.753)^2} = \mathbf{1.18\text{ kN}}, \quad \theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.753}{-0.908} = \mathbf{-39.7°}$$

Discussion Note that the magnitude of the anchoring force is 1.18 kN, and its line of action makes –39.7° from +x direction. Negative value for F_{Rx} indicates the assumed direction is wrong.