Chapter 5: Mass, Bernoulli, and Energy Equations

Introduction

- This chapter deals with 3 equations commonly used in fluid mechanics
  - *The mass equation* is an expression of the conservation of mass principle.
  - *The Bernoulli equation* is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other.
  - *The energy equation* is a statement of the conservation of energy principle.

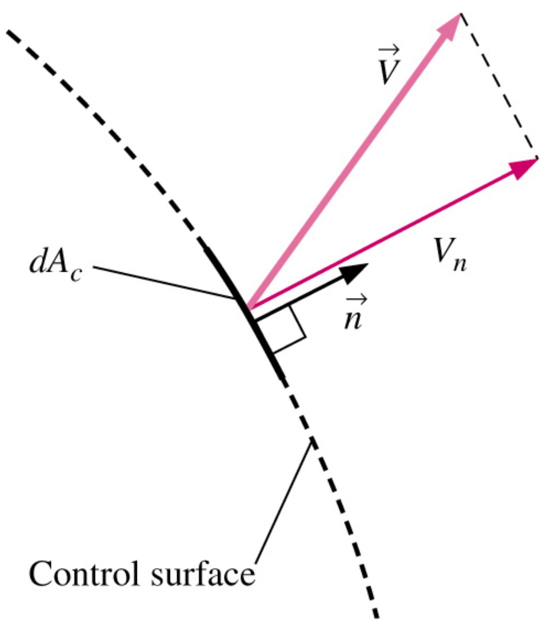
Objectives

- After completing this chapter, you should be able to
  - Apply the mass equation to balance the incoming and outgoing flow rates in a flow system.
  - Recognize various forms of mechanical energy, and work with energy conversion efficiencies.
  - Understand the use and limitations of the Bernoulli equation, and apply it to solve a variety of fluid flow problems.
  - Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.

Conservation of Mass

- Conservation of mass principle is one of the most fundamental principles in nature.
- Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.
- For *closed systems* mass conservation is implicit since the mass of the system remains constant during a process.
- For *control volumes*, mass can cross the boundaries which means that we must keep track of the amount of mass entering and leaving the control volume.

Mass and Volume Flow Rates



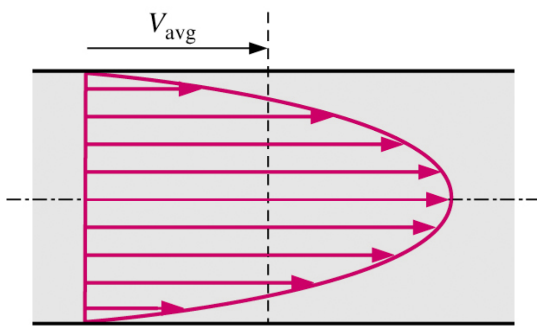
- The amount of mass flowing through a control surface per unit time is called the **mass flow rate** and is denoted $\dot{m}$
- The dot over a symbol is used to indicate *time rate of change*.
- Flow rate across the entire cross-sectional area of a pipe or duct is obtained by integration

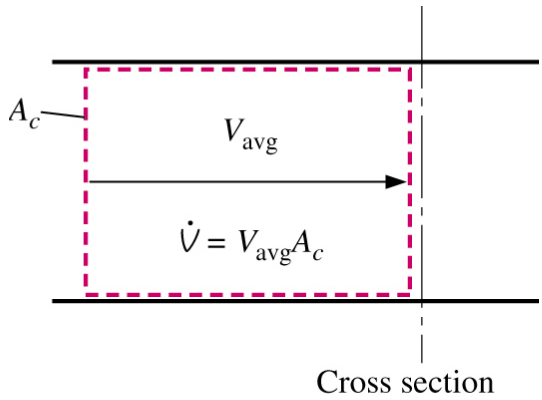
$$\dot{m} = \int_{A_c} \delta m = \int_{A_c} \rho V_n dA_c$$

- While this expression for $\dot{m}$ is exact, it is not always convenient for engineering analyses.

Note that both δ and *d are used to indicate differential quantities, but* δ *is* typically used for quantities (such as heat, work, and mass transfer)

Average Velocity and Volume Flow Rate





- Integral in $\dot{m}$ can be replaced with average values of ρ and V_n

$$V_{avg} = \frac{1}{A_c} \int_{A_c} V_n dA_c$$

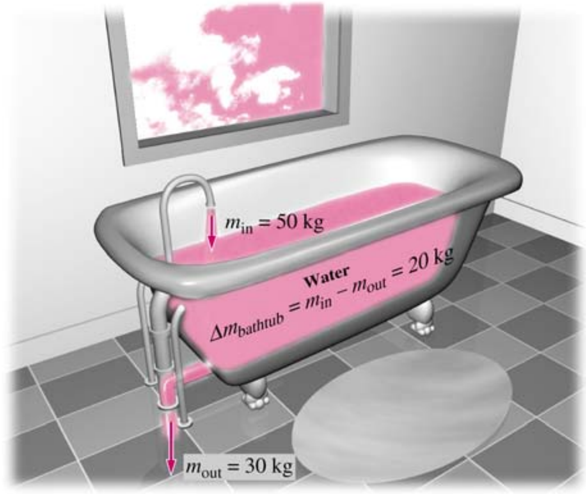
- For many flows variation of ρ is very small:
- Volume flow rate $\dot{V}$ is given by $\dot{m} = \rho V_{avg} A_c$

$$\dot{V} = \int_{A_c} V_n dA_c = V_{avg} A_c = V A_c$$

- Note: many textbooks use Q instead of $\dot{V}$ for volume flow rate.
- Mass and volume flow rates are related by

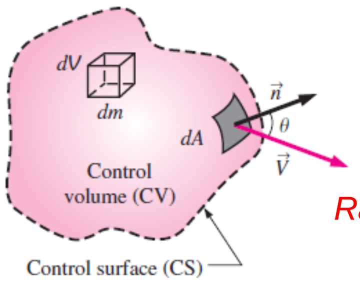
$$\dot{m} = \rho \dot{V}$$

Conservation of Mass Principle



- The **conservation of mass principle** can be expressed as
- $$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt} \quad \text{(kg/s)}$$
- Where $\dot{m}_{in}$ and $\dot{m}_{out}$ are the total rates of mass flow into and out of the CV, and dm_{CV}/dt is the rate of change of mass within the CV.

Conservation of Mass Principle



Total mass within the CV: $$m_{cv} = \int_{cv} \rho d\forall$$

Rate of change of mass within the CV: $$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho d\forall$$

Normal component of velocity: $$V_n = V\cos\theta = \vec{V}\cdot\vec{n}$$

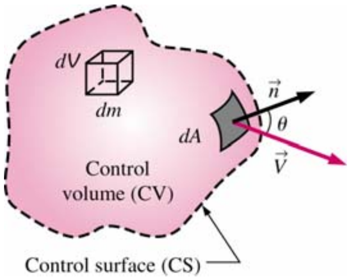
Differential mass flow rate: $$\partial\dot{m} = \rho V_n dA = \rho\ (V\cos\theta)dA = \rho\left(\vec{V}\cdot\vec{n}\right)dA$$

Net mass flow rate : $$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho\left(\vec{V}\bullet\vec{n}\right)dA$$

$$\vec{V}.\vec{n} = V\cos\theta > 0 \quad for \quad \theta < 90^o\ (outflow)$$

$$\vec{V}.\vec{n} = V\cos\theta < 0 \quad for \quad \theta > 90^o\ \text{(inflow)}$$

Conservation of Mass Principle



- For CV of arbitrary shape,
  - rate of change of mass within the CV

$$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho d\mathcal{V}$$

  - net mass flow rate

$$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho\left(\vec{V}\bullet\vec{n}\right)dA$$

- Therefore, general conservation of mass for a fixed CV is:

$$\frac{d}{dt}\int_{CV} \rho d\mathcal{V} + \int_{CS} \rho\left(\vec{V}\bullet\vec{n}\right)dA = 0$$

It states that *the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.*

Conservation of Mass Principle

Reynolds Transport Theorem b=B/m =m/m=1

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, d\mathcal{V} + \int_{CS} \rho b(\vec{V}\cdot\vec{n})\, dA$$

$$B = m \qquad b = 1 \qquad b = 1$$

$$\frac{dm_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \, d\mathcal{V} + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA$$

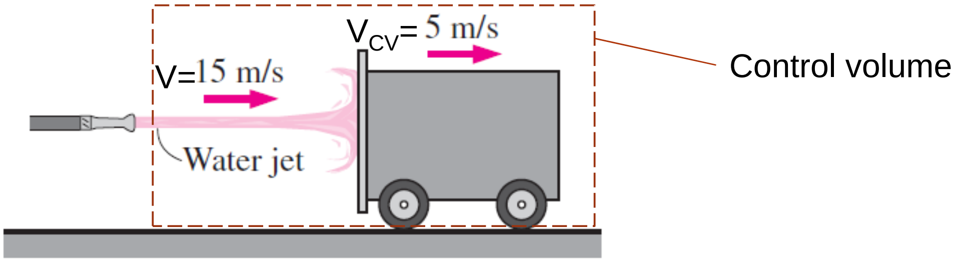
The mass of a system is constant, and thus its time derivative is zero. $\dfrac{dm_{sys}}{dt} = 0$

Conservation of Mass Principle

Moving or Deforming Control Volumes

$$\frac{d}{dt}\int_{CV}\rho\, d\mathcal{V} + \int_{CS}\rho\left(\vec{V}_r \bullet \vec{n}\right)dA = 0 \qquad \vec{V}_r = \vec{V} - \vec{V}_{CV}$$

Velocity of fluid

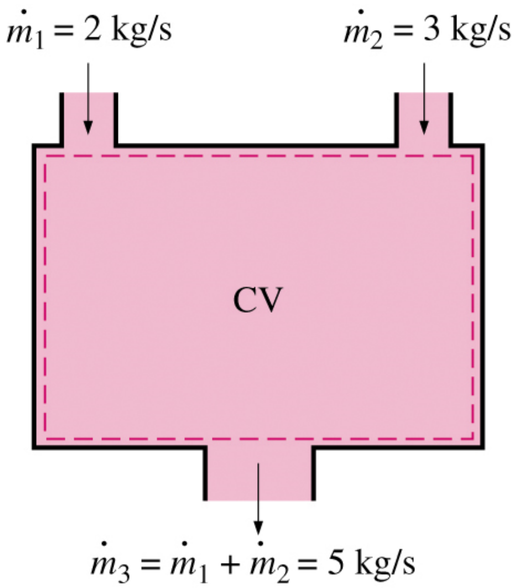


Conservation of Mass Principle

The general conservation of mass relation can also be expressed as

$$\frac{d}{dt}\int_{CV}\rho \; d\mathcal{V} = \sum_{in}\dot{m} - \sum_{out}\dot{m} \quad or \quad \frac{dm_{cv}}{dt} = \sum_{in}\dot{m} - \sum_{out}\dot{m}$$

Steady—Flow Processes



- For steady flow, the total amount of mass contained in CV is constant.
- Total amount of mass entering must be equal to total amount of mass leaving

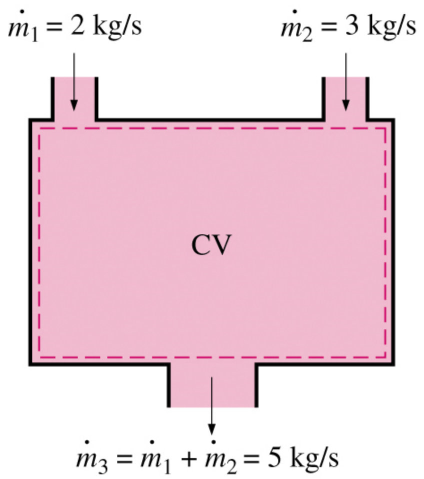
$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad \text{(kg/s)}$$

$$\sum_{in} \dot{V} = \sum_{out} \dot{V} \quad (m^3/s)$$

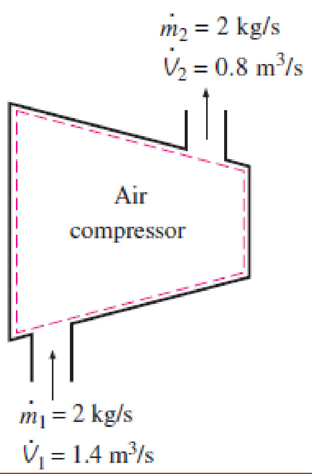
Special Case: Incompressible Flow

Steady, incompressible flow:

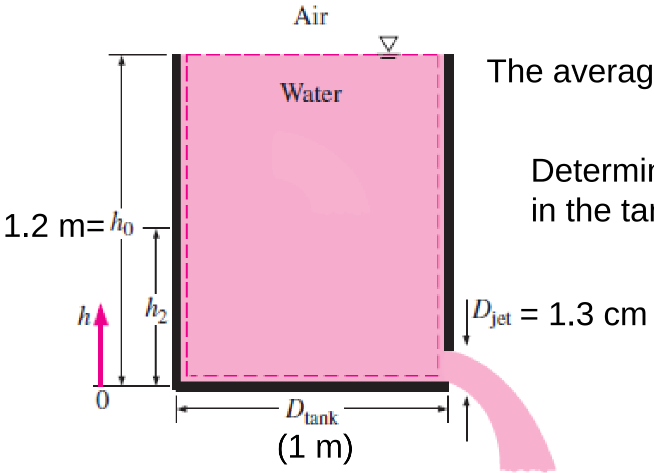
$$\sum_{in} \dot{V} = \sum_{out} \dot{V} \quad (m^3/s)$$



Steady, compressible flow



EXAMPLE - Discharge of Water from a Tank



The average velocity of the jet is given by $V = \sqrt{2gh}$

Determine how long it will take for the water level in the tank to drop to h=0.6 m from the bottom.

$$\underbrace{\dot{m}_{in}}_{0} - \dot{m}_{out} = \frac{dm_{cv}}{dt}$$

$$\dot{m}_{out} = (\rho VA)_{out} = \rho\sqrt{2gh}A_{jet}$$

$$m_{CV} = \rho \cancel{V} = \rho A_{\tan k} h$$

$$-\rho\sqrt{2gh}A_{jet} = \frac{d(\rho A_{\tan k} h)}{dt} \rightarrow \quad -\rho\sqrt{2gh}(\pi D^2{}_{jet}/4) = \frac{\rho(\pi D^2{}_{\tan k}/4)dh}{dt}$$

$$dt = -\frac{D^2{}_{\tan k}}{D^2{}_{jet}}\frac{dh}{\sqrt{2gh}}$$

$$\int_0^t dt = \frac{D^2{}_{\tan k}}{D^2{}_{jet}\sqrt{2g}}\int_{h_0}^{h_2}\frac{dh}{\sqrt{h}} \quad \rightarrow \quad t = \frac{\sqrt{h_0}-\sqrt{h_2}}{\sqrt{g/2}}\left(\frac{D_{\tan k}}{D_{jet}}\right)^2$$

t = 857 s = 14.3 min (h_2=0.6 m)

t = 48.8 min (h_2=0 m)

Mechanical Energy

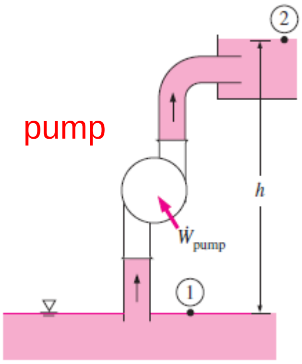
- **Mechanical energy** can be defined as *the form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device such as an ideal turbine.*
- Flow P/ρ, kinetic V^2/g, and potential gz energy are the forms of mechanical energy $e_{mech} = P/\rho + V^2/g + gz$ (J/kg)
- Mechanical energy change of a fluid during incompressible flow becomes

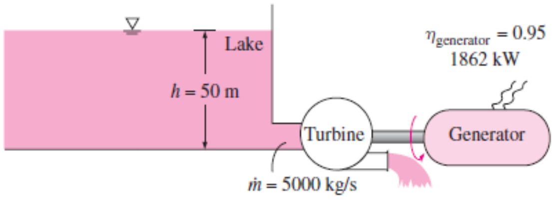
$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad \text{(J/kg)}$$

- In the absence of loses, Δe_{mech} represents the work supplied to the fluid (Δe_{mech}>0) or extracted from the fluid (Δe_{mech}<0).

Mechanical Energy

A pump transfers mechanical energy to a fluid,
and
a turbine extracts mechanical energy from a fluid.





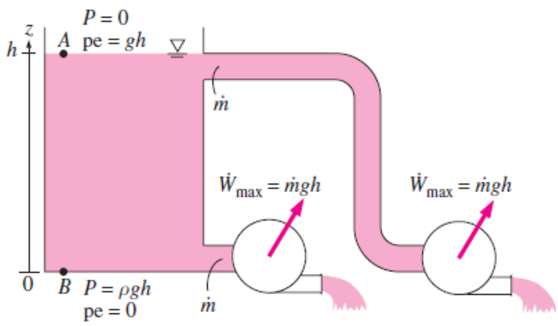
turbine

For an ideal hydraulic turbine $w_{turbine} = gh$ *(j/kg)*

➢Note that we are also assuming ideal flow (no irreversible losses) through the pipe leading from the tank to the turbine.

➢Therefore, the total mechanical energy of water at the bottom is equivalent to that at the top.

Mechanical Energy



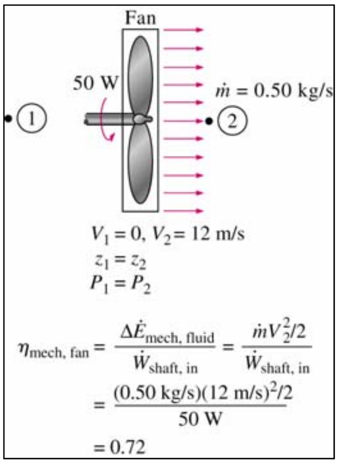
The mechanical energy of water at the bottom of a container is equal to the mechanical energy at any depth including the free surface of the container.

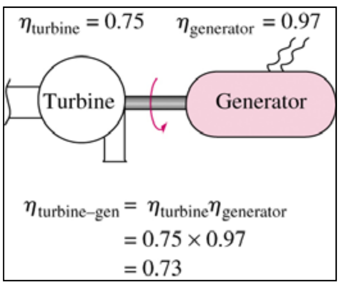
Mechanical Efficiency

- Transfer of e_{mech} is usually accomplished by a rotating shaft: *shaft work*
- Pump, fan, propulsion: receives shaft work (e.g., from an electric motor) and transfers it to the fluid as mechanical energy
- Turbine: converts e_{mech} of a fluid to shaft work.
- If η_{mech} < 100%, losses have occurred during conversion.

$$\eta_{mech} = \frac{E_{mech,out}}{E_{mech,in}} = 1 - \frac{E_{mech,loss}}{E_{mech,in}}$$

Pump and Turbine Efficiencies





- In fluid systems, we are usually interested in increasing the pressure, velocity, and/or elevation of a fluid.
- In these cases, efficiency is better defined as the ratio of (supplied or extracted work) vs. rate of increase in mechanical energy

$$\eta_{pump} = \frac{\text{Mechanical energy increase of the fluid}}{\text{Mechanical energy input}} = \frac{\Delta E_{mech,\ fluid}}{\dot{W}_{shaft,in}} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}}$$

$$\eta_{turbine} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy decrease of the fluid}} = \frac{\dot{W}_{shaft,out}}{\left|\Delta \dot{E}_{mech,\ fluid}\right|} = \frac{\dot{W}_{turbine}}{\dot{W}_{turbine,e}}$$

- Overall efficiency must include motor or generator efficiency.

Mechanical Efficiency

The mechanical efficiency should not be confused with the **motor efficiency and the generator efficiency.**

$$\eta_{motor} = \frac{Mechanical\ power\ output}{Electric\ power\ input} = \frac{\dot{W}_{shaft,out}}{\dot{W}_{elect,in}}$$

$$\eta_{generator} = \frac{Electric\ power\ output}{Mechanical\ power\ input} = \frac{\dot{W}_{elect,out}}{\dot{W}_{shaft,in}}$$

A pump is usually packaged together with its motor, and a turbine with its generator.

The combined or overall efficiency of pump–motor and turbine–generator combinations

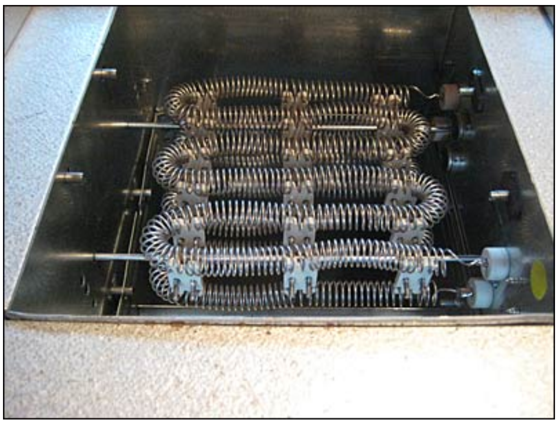
$$\eta_{pump-motor} = \eta_{pump}\eta_{motor} = \frac{\dot{W}_{pump,u}}{\dot{W}_{elect,in}} = \frac{\Delta\dot{E}_{mech,\ fluid}}{W_{elec,in}}$$

$$\eta_{turbine-gen} = \eta_{turbine}\eta_{generator} = \frac{\dot{W}_{elect,out}}{\dot{W}_{turbine,e}} = \frac{\dot{W}_{elect,out}}{\left|\Delta\dot{E}_{mech,\ fluid}\right|}$$

Mechanic Efficiency

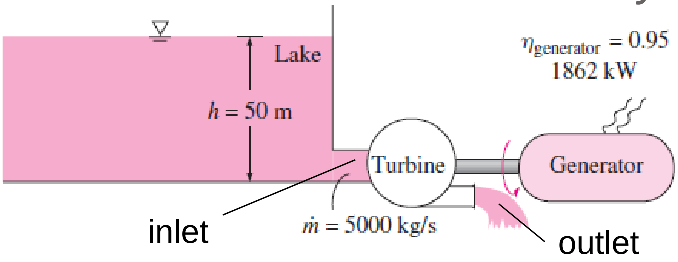
The lower limit of 0 percent corresponds to the conversion of the entire mechanical or electric energy input to thermal energy, and the device in this case functions like a resistance heater.

$$\eta_{mechanic}\ \text{of a resistance heater is 0}$$



A typical resistance heater

EXAMPLE - Performance of a Hydraulic Turbine–Generator



Assumptions

1) The elevation of the lake remains constant.

2) The mechanical energy of water at the turbine exit is negligible.

$$e_{mech,in} - e_{mech,out} = \frac{P}{\rho} - 0 = gh = \left(9.81\ \mathrm{m/s^2}\right)\left(50m\right) = 491\ \mathrm{m^2/s^2} = 491\ \mathrm{j/kg}$$

➢The rate at which mechanical energy is supplied to the turbine by the fluid and the overall efficiency become :

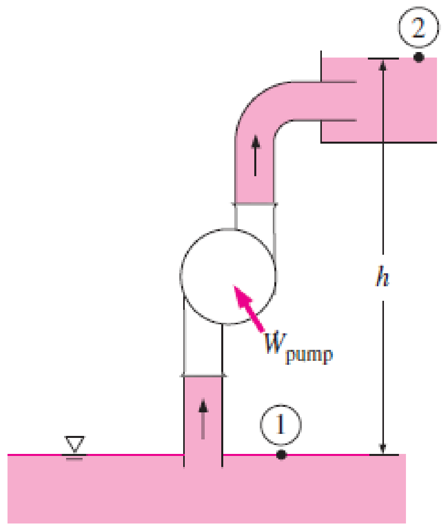
$$\left|\Delta \dot{E}_{mech,\,fluid}\right| = \dot{m}\left(e_{mech,in} - e_{mech,out}\right) = \left(5000\ \mathrm{kg/s}\right)\left(0.491\ \mathrm{kj/kg}\right) = 2455\ \mathrm{kW}$$

$$\eta_{overall} = \eta_{turbine-gen} = \frac{\dot{W}_{elect,out}}{\left|\Delta \dot{E}_{mech,\,fluid}\right|} = \frac{1862\ \mathrm{kW}}{2455\ \mathrm{kW}} = 0.76$$

$$\eta_{turbine-gen} = \eta_{turbine}\eta_{generator} \rightarrow \eta_{turbine} = \frac{\eta_{turbine-gen}}{\eta_{generator}} = \frac{0.76}{0.95} = 0.80$$

$$\dot{W}_{shaft,out} = \eta_{turbine}\left|\Delta \dot{E}_{mech,\,fluid}\right| = \left(0.80\right)\left(2455\ \mathrm{kW}\right) = 1964\ \mathrm{kW}$$

Mechanical energy balance



Steady flow

$$V_1 = V_2$$
$$z_2 = z_1 + h$$
$$P_1 = P_2 = P_{atm}$$

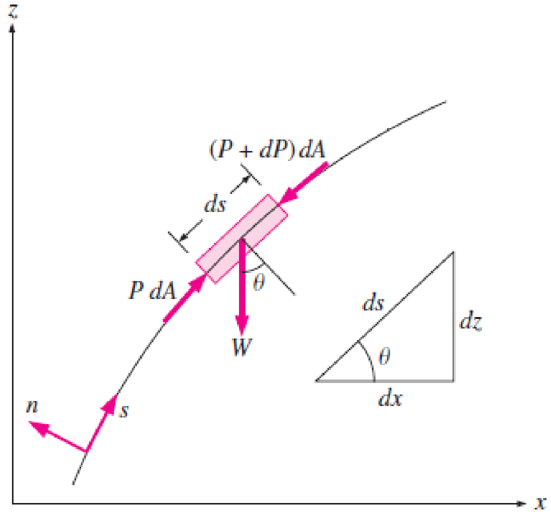
$$\dot{E}_{mech,\,in} = \dot{E}_{mech,\,out} + \dot{E}_{mech,\,loss}$$
$$\dot{W}_{pump} + \dot{m}gz_1 = \dot{m}gz_2 + \dot{E}_{mech,\,loss}$$
$$\dot{W}_{pump} = \dot{m}gh + \dot{E}_{mech,\,loss}$$

Most fluid flow problems involve mechanical forms of energy only, and such problems are conveniently solved by using a *mechanical energy balance.*

Steady process :

$$E_{mech,in} - E_{mech,out} = \Delta E_{mech,loss}$$

Bernoulli Equation



The forces acting on a fluid particle along a streamline.

Applying the *conservation of linear momentum principle (Newton's 2nd law)*

<u>Assumption:</u> Viscous effects are negligibly small compared to inertial, gravitational, and pressure effects.

$$\sum F_s = ma_s$$

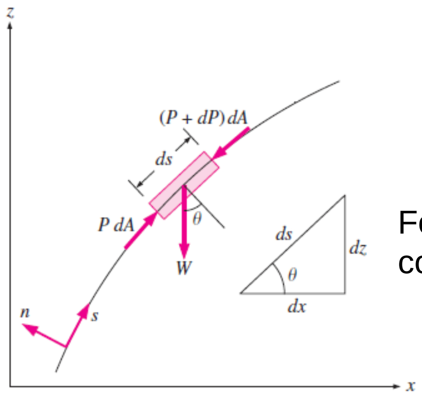
$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$$

$$PdA - (P+dP)dA - W\sin\theta = m\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}\right)$$

$$-dPdA - \rho g dA ds\frac{dz}{ds} = \rho dA ds\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}\right)$$

$$-dP - \rho g dz = \rho\left(\frac{\partial V}{\partial t}ds + VdV\right) \Rightarrow \frac{dP}{\rho} + \frac{1}{2}d(V^2) + gdz + \frac{\partial V}{\partial t}ds = 0$$

Bernoulli Equation



$$\frac{dP}{\rho} + \frac{1}{2}d(V^2) + gdz + \frac{\partial V}{\partial t}ds = 0$$

For unsteady, compressible flow: $$\int\frac{dP}{\rho} + \frac{V^2}{2} + gz + \int\frac{\partial V}{\partial t}ds = \text{constant}$$

(along a streamline)

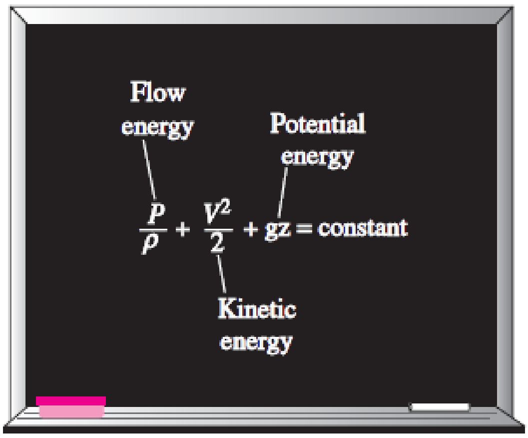
Steady, incompressible flow: $$\boxed{\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}}$$

This is the famous **Bernoulli Equation,** which is commonly used in fluid mechanics for *steady*, *incompressible flow along a streamline in inviscid regions of flow*.

<u>Between any two points on the same streamline:</u>

Steady, incompressible flow: $$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Bernoulli Equation

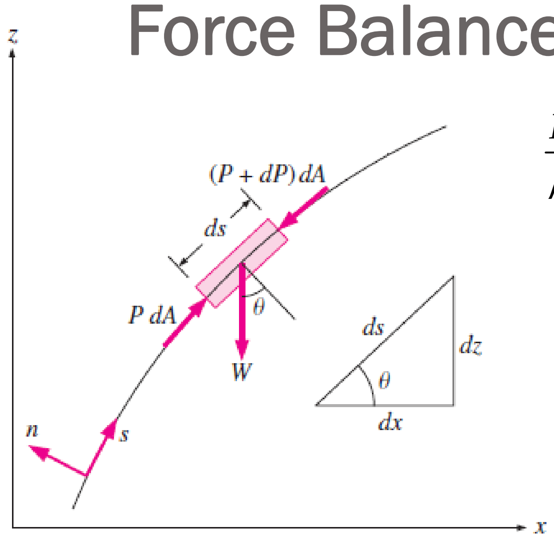


- Bernoulli equation can be viewed as an expression of *mechanical energy balance* *and can be stated as follows:*

The sum of the *kinetic, potential*, and *flow energies* of a fluid particle is constant along a streamline during steady flow *when the compressibility and frictional effects are negligible.*

The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.

Force Balance across Streamlines

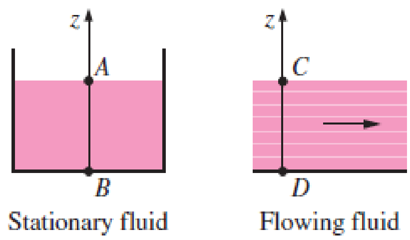


$$\frac{P}{\rho} + \int \frac{V^2}{R} dn + gz = \text{constant} \qquad \text{(across streamlines)}$$

$$\text{As } R \to \infty \qquad \frac{P}{\rho} + gz = \text{constant}$$

$$P = -\rho gz + \text{constant}$$

The variation of pressure with elevation in steady, incompressible flow along a straight line is the same as that in the stationary fluid (but this is not the case for a curved flow section).



$$P_B - P_A = P_D - P_C$$

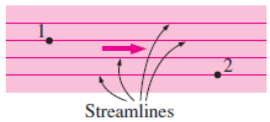
Limitations on the Use of the Bernoulli Equation

Bernoulli Equation: $\dfrac{P}{\rho}+\dfrac{V^2}{2}+gz=\text{constant}$ or $\dfrac{P_1}{\rho}+\dfrac{V_1^{\,2}}{2}+gz_1=\dfrac{P_2}{\rho}+\dfrac{V_2^{\,2}}{2}+gz_2$

1) Steady flow
2) Frictionless flow
3) No shaft work ($w_{pump}=w_{turbine}=0$)
4) Incompressible flow

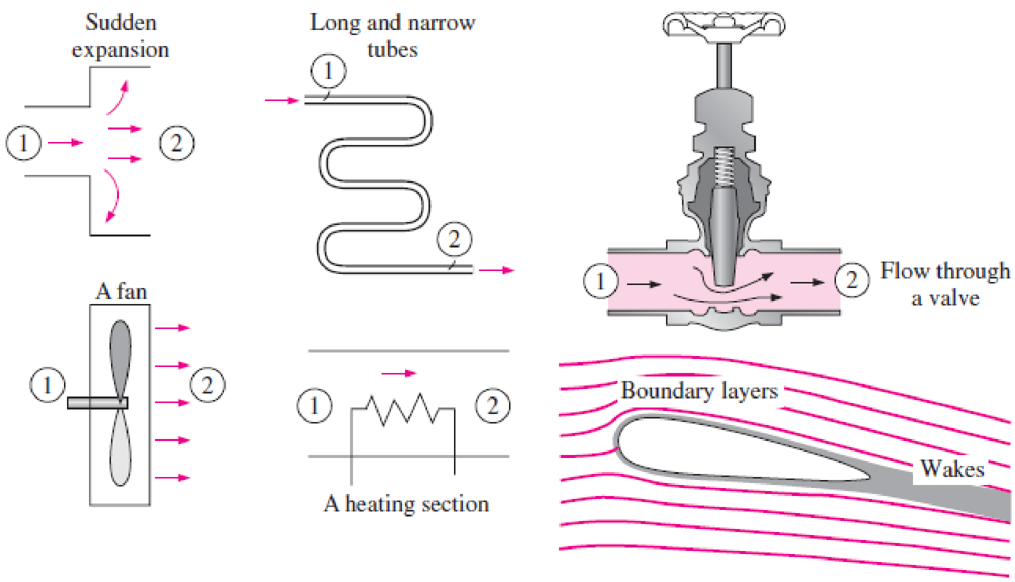
$$\textit{if Mach } \text{number}=\frac{V}{V_{sound}}<0.3 \quad \text{(density variation is negligible)}$$

5) No heat transfer (The Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections)
6) Flow along a streamline or irrotational flow



Limitations on the Use of the Bernoulli Equation

Bernoulli Equation: $\dfrac{P}{\rho}+\dfrac{V^2}{2}+gz=\text{constant}$



Frictional effects and components that disturb the streamlined structure of flow in a flow section make the Bernoulli equation invalid.

Static, Dynamic, and Stagnation Pressures

Bernoulli Equation : $P + \rho\frac{V^2}{2} + \rho gz = \text{constant} = \text{Total pressure}$

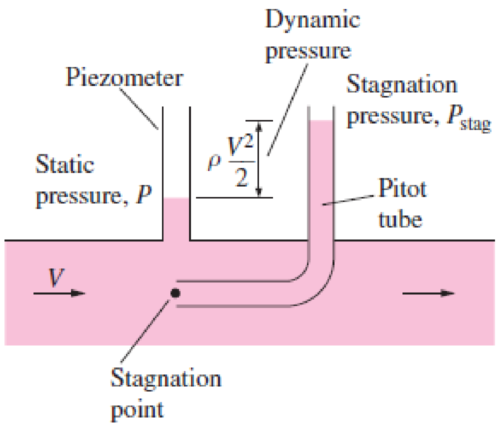
*The **static pressure P (it does not incorporate any dynamic effects); it*** represents the **actual thermodynamic pressure** of the fluid.

$\rho V^2/2$ *is the **dynamic pressure;** it represents the pressure rise when the* fluid in motion is brought to a **stop isentropically.**

ρgz accounts for the elevation effects, i.e., of fluid weight on pressure.

The Bernoulli equation states that *the total pressure along a streamline is constant.*

Static, Dynamic, and Stagnation Pressures



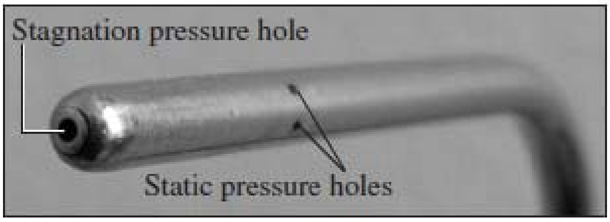
$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

The sum of the static and dynamic pressures is called the **stagnation pressure.**

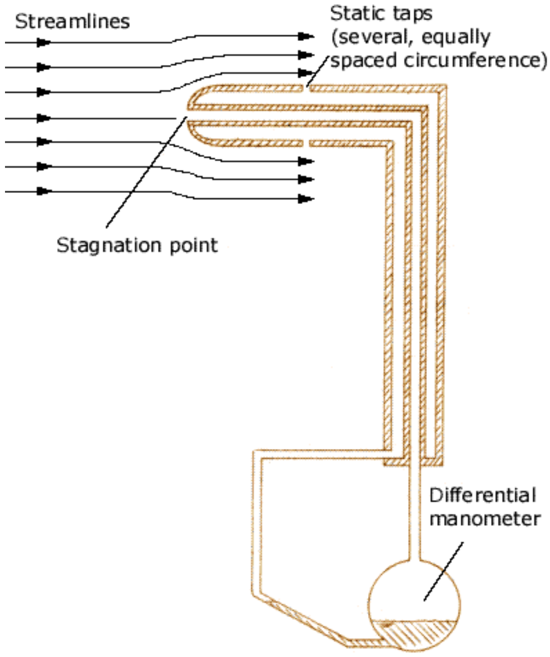
$$P_{stag} = P + \rho\frac{V^2}{2} \quad (Pa)$$

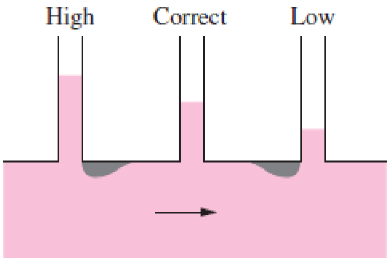
The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically.

Static, Dynamic, and Stagnation Pressures - Pitot Tube



$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

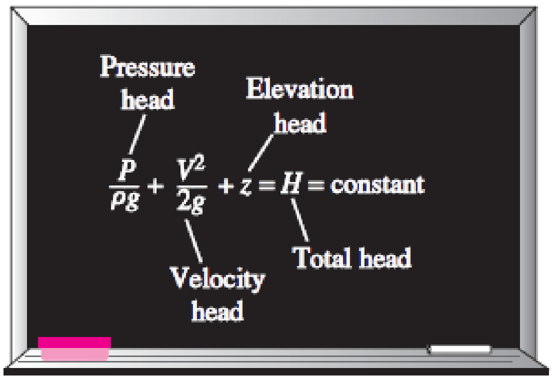




Careless drilling of the static pressure tap may result in an erroneous reading of the static pressure.

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant} \quad \text{(along a streamline)}$$

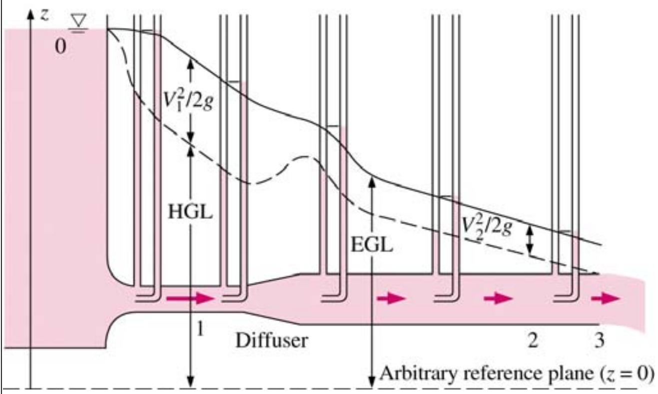


➢*P/ρg is the* ***pressure head; it represents the height of a fluid column that*** produces the static pressure *P.*

➢*V²/2g is the* ***velocity head; it represents the elevation needed for a fluid*** to reach the velocity *V during frictionless free fall.*

➢*z is the* ***elevation head; it represents the potential energy of the fluid.***

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)



- It is often convenient to plot mechanical energy graphically using heights.
- Hydraulic Grade Line

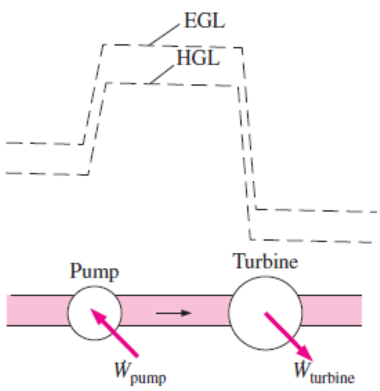
$$HGL = \frac{P}{\rho g} + z$$

- Energy Grade Line (or total energy)

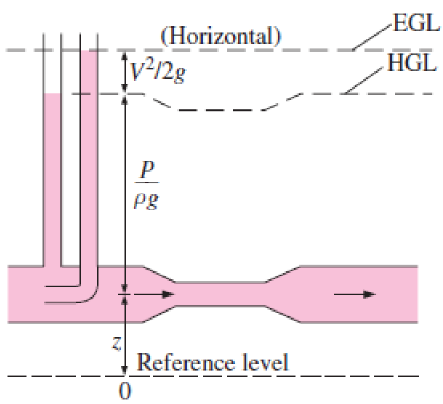
$$EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

- ➢ The EGL is always a distance *$V^2/2g$ above the HGL.*
- ➢ For *open-channel flow, the HGL coincides with the free surface of the* liquid, and the EGL is a distance *$V^2/2g$ above the free surface.*

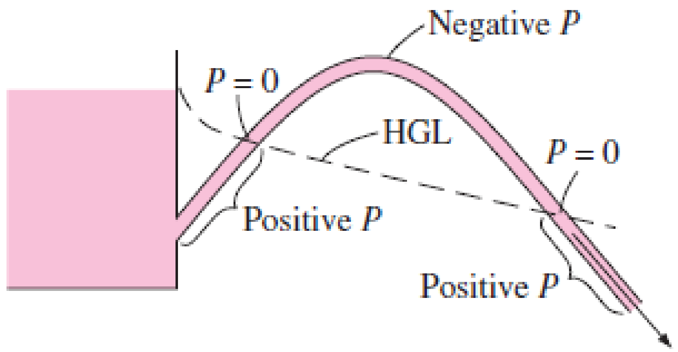


A *steep jump occurs in EGL and HGL* whenever mechanical energy is added to the fluid by a pump, and a *steep drop* occurs whenever mechanical energy is removed from the fluid by a turbine.



In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. But this is not the case for HGL when the flow velocity varies along the flow.

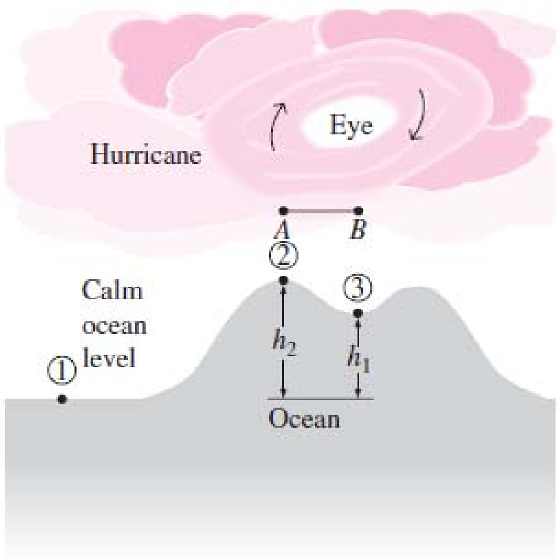
Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)



➢ EGL declines continually along the flow direction due to friction and other irreversible losses in the flow.

➢ EGL cannot increase in the flow direction unless energy is supplied to the fluid.

The Rise of the Ocean Due to a Hurricane



A hurricane is a tropical storm formed over the ocean by low atmospheric pressures. As a hurricane approaches land, inordinate ocean swells (very high tides) accompany the hurricane. A Class-5 hurricane features winds in excess of 155 mph, although the wind velocity at the center "eye" is very low.

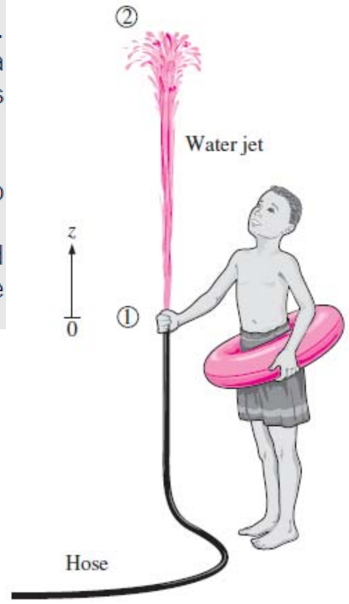
The wind power of hurricanes is not the only cause of damage to coastal areas. Ocean flooding and erosion from excessive tides is just as serious, as are high waves generated by the storm turbulence and energy.

Examples

EXAMPLE 5–5 Spraying Water into the Air

Water is flowing from a hose attached to a water main at 400 kPa gage (Fig. 5–38). A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. If the hose is held upward, what is the maximum height that the jet could achieve?

SOLUTION Water from a hose attached to the water main is sprayed into the air. The maximum height the water jet can rise is to be determined.
Assumptions **1** The flow exiting into the air is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The water pressure in the hose near the outlet is equal to the water main pressure. **3** The surface tension effects are negligible. **4** The friction between the water and air is negligible. **5** The irreversibilities that may occur at the outlet of the hose due to abrupt expansion are negligible.
Properties We take the density of water to be 1000 kg/m^3.



Examples

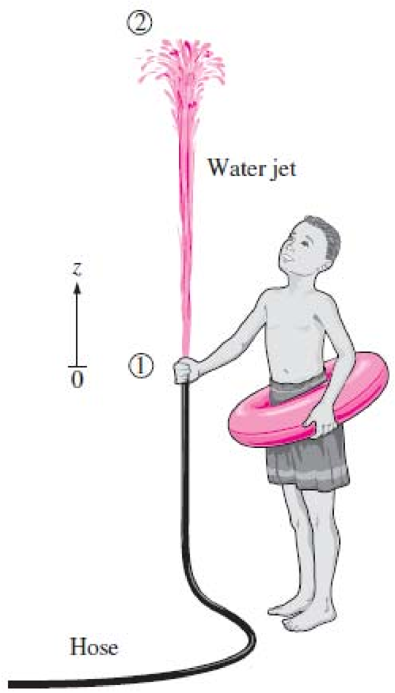
Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1 \cong 0$) and we take the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \cancelto{0}{\frac{V_1^2}{2g}} + \cancelto{0}{z_1} = \frac{P_2}{\rho g} + \cancelto{0}{\frac{V_2^2}{2g}} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} + z_2$$

Solving for z_2 and substituting,

$$z_2 = \frac{P_1 - P_{atm}}{\rho g} = \frac{P_{1,\,gage}}{\rho g} = \frac{400 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)$$
$$= \mathbf{40.8 \text{ m}}$$

Therefore, the water jet can rise as high as 40.8 m into the sky in this case.
Discussion The result obtained by the Bernoulli equation represents the upper limit and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.8 m, and, in all likelihood, the rise will be much less than 40.8 m due to irreversible losses that we neglected.



Examples

EXAMPLE 5–6 Water Discharge from a Large Tank

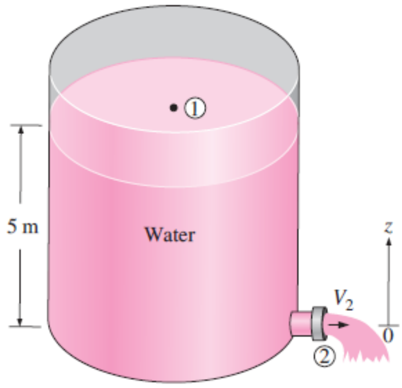
A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap (Fig. 5–39). A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the water velocity at the outlet.

SOLUTION A tap near the bottom of a tank is opened. The exit velocity of water from the tank is to be determined.

Assumptions **1** The flow is incompressible and irrotational (except very close to the walls). **2** The water drains slowly enough that the flow can be approximated as steady (actually quasi-steady when the tank begins to drain).

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of water so that $P_1 = P_{atm}$ (open to the atmosphere), $V_1 \cong 0$ (the tank is large relative to the outlet), and $z_1 = 5$ m and $z_2 = 0$ (we take the reference level at the center of the outlet). Also, $P_2 = P_{atm}$ (water discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g}^{\nearrow 0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + z_2^{\nearrow 0} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$



Examples

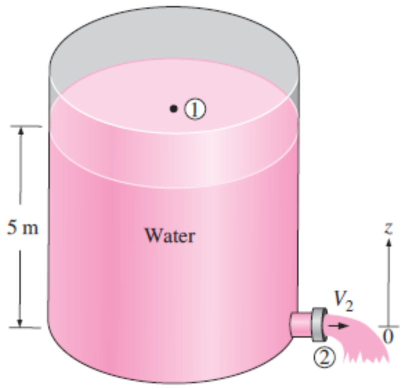
Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81 \text{ m/s}^2)(5 \text{ m})} = \mathbf{9.9 \text{ m/s}}$$

The relation $V = \sqrt{2gz}$ is called the **Toricelli equation.**

Therefore, the water leaves the tank with an initial velocity of 9.9 m/s. This is the same velocity that would manifest if a solid were dropped a distance of 5 m in the absence of air friction drag. (What would the velocity be if the tap were at the bottom of the tank instead of on the side?)

Discussion If the orifice were sharp-edged instead of rounded, then the flow would be disturbed, and the velocity would be less than 9.9 m/s, especially near the edges. Care must be exercised when attempting to apply the Bernoulli equation to situations where abrupt expansions or contractions occur since the friction and flow disturbance in such cases may not be negligible.



Examples

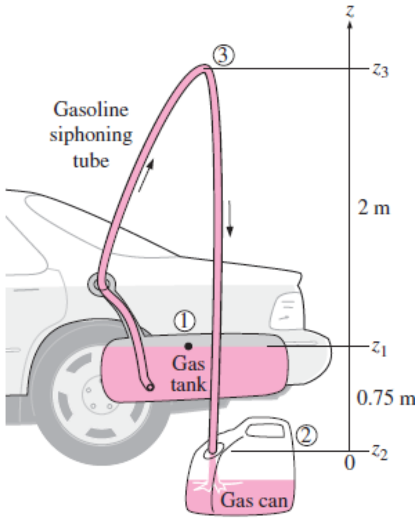
EXAMPLE 5–7 Siphoning Out Gasoline from a Fuel Tank

During a trip to the beach (P_{atm} = 1 atm = 101.3 kPa), a car runs out of gasoline, and it becomes necessary to siphon gas out of the car of a Good Samaritan (Fig. 5–40). The siphon is a small-diameter hose, and to start the siphon it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) causes the liquid to flow from the higher to the lower elevation. Point 2 is located 0.75 m below point 1 in this case, and point 3 is located 2 m above point 1. The siphon diameter is 4 mm, and frictional losses in the siphon are to be disregarded. Determine (*a*) the minimum time to withdraw 4 L of gasoline from the tank to the can and (*b*) the pressure at point 3. The density of gasoline is 750 kg/m³.

SOLUTION Gasoline is to be siphoned from a tank. The minimum time it takes to withdraw 4 L of gasoline and the pressure at the highest point in the system are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Even though the Bernoulli equation is not valid through the pipe because of frictional losses, we employ the Bernoulli equation anyway in order to obtain a *best-case estimate*. **3** The change in the gasoline surface level inside the tank is negligible compared to elevations z_1 and z_2 during the siphoning period.

Properties The density of gasoline is given to be 750 kg/m³.



Analysis (*a*) We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere), $V_1 \cong 0$ (the tank is large relative to the tube diameter), and $z_2 = 0$ (point 2 is taken as the reference level). Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \underbrace{\frac{V_1^2}{2g}}_{\nearrow 0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \underbrace{z_2}_{\nearrow 0} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(0.75\ \text{m})} = 3.84\ \text{m/s}$$

The cross-sectional area of the tube and the flow rate of gasoline are

$$A = \pi D^2/4 = \pi(5 \times 10^{-3}\ \text{m})^2/4 = 1.96 \times 10^{-5}\ \text{m}^2$$

$$\dot{V} = V_2 A = (3.84\ \text{m/s})(1.96 \times 10^{-5}\ \text{m}^2) = 7.53 \times 10^{-5}\ \text{m}^3/\text{s} = 0.0753\ \text{L/s}$$

Then the time needed to siphon 4 L of gasoline becomes

$$\Delta t = \frac{V}{\dot{V}} = \frac{4\ \text{L}}{0.0753\ \text{L/s}} = \mathbf{53.1\ s}$$

(*b*) The pressure at point 3 can be determined by writing the Bernoulli equation between points 2 and 3. Noting that $V_2 = V_3$ (conservation of mass), $z_2 = 0$, and $P_2 = P_{atm}$,

$$\frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \underbrace{z_2}_{\nearrow 0} = \frac{P_3}{\rho g} + \cancel{\frac{V_3^2}{2g}} + z_3 \quad \rightarrow \quad \frac{P_{atm}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

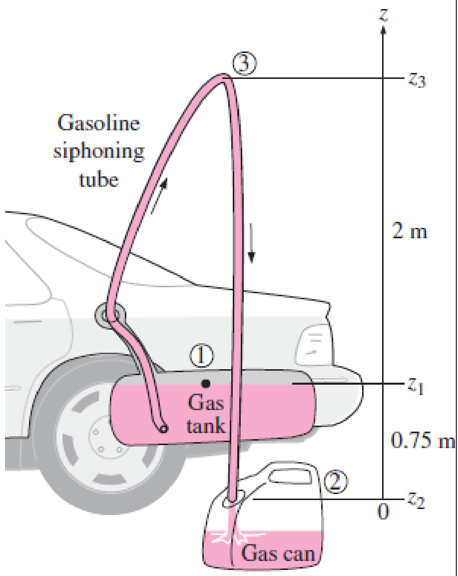
Solving for P_3 and substituting,

$$P_3 = P_{atm} - \rho g z_3$$

$$= 101.3\ \text{kPa} - (750\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.75\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

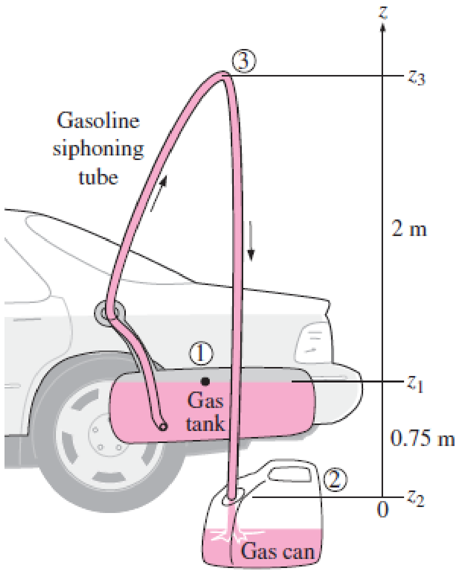
$$= \mathbf{81.1\ kPa}$$

Examples



Examples

Discussion The siphoning time is determined by neglecting frictional effects, and thus this is the *minimum time* required. In reality, the time will be longer than 53.1 s because of friction between the gasoline and the tube surface. Also, the pressure at point 3 is below the atmospheric pressure. If the elevation difference between points 1 and 3 is too high, the pressure at point 3 may drop below the vapor pressure of gasoline at the gasoline temperature, and some gasoline may evaporate (cavitate). The vapor then may form a pocket at the top and halt the flow of gasoline.



EXAMPLE 5–8 Velocity Measurement by a Pitot Tube

A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in Fig. 5–41, to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.

SOLUTION The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined.
Assumptions 1 The flow is steady and incompressible. 2 Points 1 and 2 are close enough together that the irreversible energy loss between these two points is negligible, and thus we can use the Bernoulli equation.
Analysis We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip of the Pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$$P_1 = \rho g(h_1 + h_2)$$

$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}^{\nearrow 0} + \cancel{z_2} \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for V_1 and substituting,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81 \text{ m/s}^2)(0.12 \text{ m})} = \mathbf{1.53 \text{ m/s}}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube.

Examples