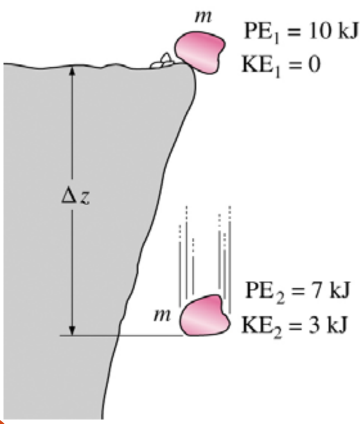
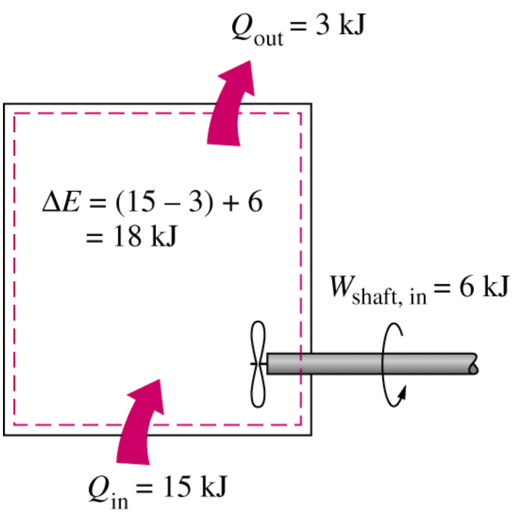
General Energy Equation

- One of the most fundamental laws in nature is the **1st law of thermodynamics**, which is also known as the **conservation of energy principle**.
- It states that *energy can be neither created nor destroyed during a process; it can only change forms*



- Falling rock, picks up speed as PE is converted to KE.
- If air resistance is neglected, PE + KE = constant

General Energy Equation



- The energy content of a closed system can be changed by two mechanisms: *heat transfer Q* and *work transfer W*.
- Conservation of energy for a closed system can be expressed in rate form as

$$\dot{Q}_{net,in} + \dot{W}_{net,in} = \frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{sys} \rho e d\mathcal{V}$$

- Net rate of heat transfer to the system:

$$\dot{Q}_{net,in} = \dot{Q}_{in} - \dot{Q}_{out}$$

- Net power input to the system:

$$\dot{W}_{net,in} = \dot{W}_{in} - \dot{W}_{out}$$

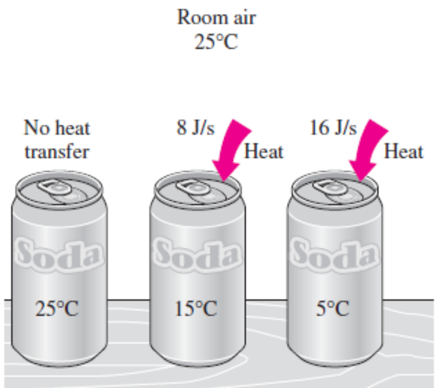
General Energy Equation

➢ For simple compressible systems, total energy consists of internal, kinetic, and potential energies.

$$e = u + ke + pe = u + \frac{V^2}{2} + gz \qquad \text{(j/kg)}$$

Energy Transfer by Heat, Q

Thermal energy tends to move naturally in the direction of decreasing temperature, and the transfer of thermal energy from one system to another as a result of a temperature difference is called **heat transfer.**



General Energy Equation

A process during which there is no heat transfer is called an **adiabatic process.**

There are two ways a process can be adiabatic:

1) **Either the system is** well insulated so that only a negligible amount of heat can pass through the system boundary,

2) or both the system and the surroundings are at the same temperature and therefore there is no driving force (temperature difference) for heat transfer.

An adiabatic process should not be confused with an isothermal process.

Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work transfer.

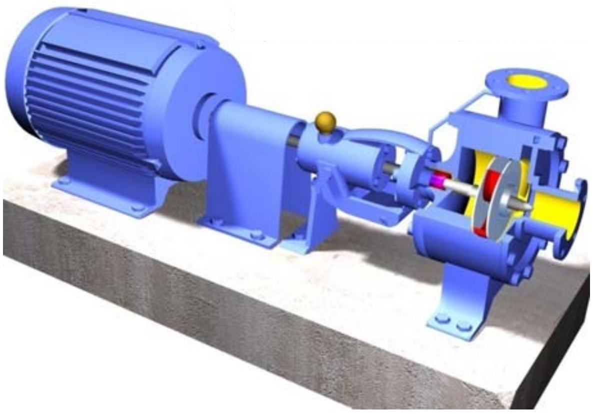
Energy Transfer by Work, *W*

➢An energy interaction is **work if it is associated with a force acting through** a distance.

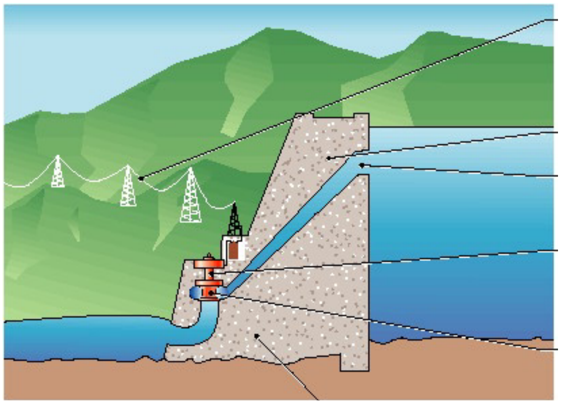
➢The time rate of doing work is called **power and is denoted by** $\dot{W}$

Car engines and hydraulic, steam, and gas turbines produce work;

Compressors, pumps, fans, and mixers consume work.



PUMP



HYDRAULIC TURBINE

Energy Transfer by Work, *W*

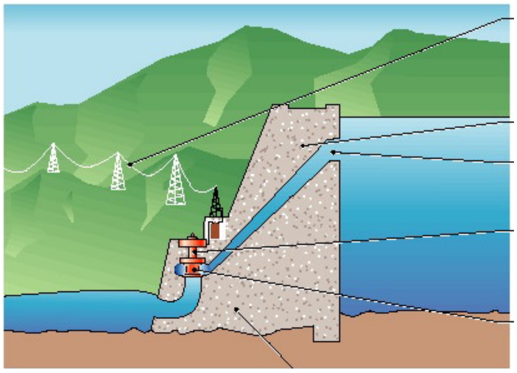
➢ Work-consuming devices transfer energy to the fluid, and thus increase the energy of the fluid.

A fan in a room, for example, mobilizes the air and increases its kinetic energy.

➢ Work-producing devices extract energy of the fluid, and thus decrease the energy of the fluid.

A hydraulic turbine in a hydraulic power plant, extracts the mechanical energy of the water in the river and decreases its kinetic energy.

FAN



HYDRAULIC TURBINE

Energy Transfer by Work, *W*

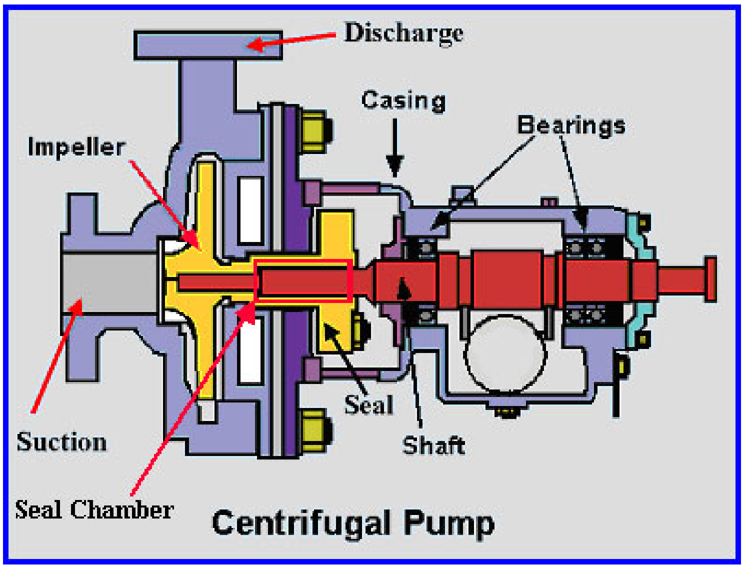
A system may involve numerous forms of work, and the total work can be expressed as

$$W_{total} = W_{shaft} + W_{pressure} + W_{viscous} + W_{other}$$

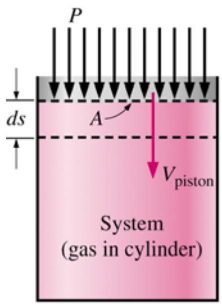
- W_{shaft} *is the work transmitted by a rotating shaft. (e.g. PUMP)*
- $W_{pressure}$ *is the work* done by the pressure forces on the control surface. (e.g. piston in a car engine cylinder)
- $W_{viscous}$ *is the work done* by the normal and shear components of viscous forces on the control surface (e.g. blades in turbomachines)
- W_{other} *is the work done by other forces such as electric, magnetic,* and surface tension

Shaft work

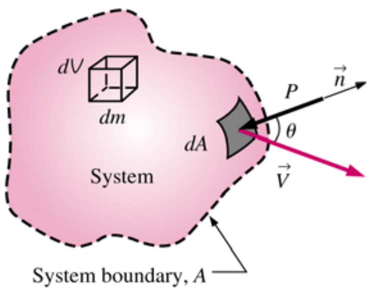
$$\dot{W}_{shaft} = \omega T_{shaft} = 2\pi \dot{n} T_{shaft}$$



Work Done by Pressure Forces



(a)



(b)

- Where does expression for pressure work come from?
- When piston moves down ds under the influence of $F=PA$, the work done on the system is $\delta W_{boundary}=PAds$.
- If we divide both sides by dt, we have

$$\delta \dot{W}_{pressure} = \delta \dot{W}_{boundary} = PA\frac{ds}{dt} = PAV_{piston}$$

- For generalized control volumes:

$$\delta \dot{W}_{pressure} = -PdAV_n = -PdA\left(\vec{V}\cdot\vec{n}\right)$$

- Note sign conventions:
  - $\vec{n}$ is outward pointing normal
  - Negative sign ensures that work done is positive when is done *on* the system.

Work Done by Pressure Forces

The total rate of work done by pressure forces is obtained by integrating $\delta W_{pressure}$ over the entire surface *A.*

$$\dot{W}_{pressure,\ net\ in} = \int \delta \dot{W}_{pressure} = -\int_A P\left(\vec{V}.\vec{n}\right)dA = -\int_A \frac{P}{\rho}\underbrace{\rho\left(\vec{V}.\vec{n}\right)dA}_{\text{mass flow through control surfaces}}$$

Energy Transfer to the system by Work and Heat

The net power transfer can be expressed as

$$\dot{W}_{net\,in} = \dot{W}_{shaft,net\,in} + \dot{W}_{pressure,net\,in} = \dot{W}_{shaft,net\,in} - \int_A P\left(\vec{V}.\vec{n}\right)dA$$

Then the rate form of the conservation of energy relation for a closed system becomes

$$\dot{Q}_{net\,in} + \dot{W}_{shaft,net\,in} + \dot{W}_{pressure,net\,in} = \frac{dE_{sys}}{dt}$$

General Energy Equation

To obtain a relation for the conservation of energy for a *control volume*, we apply the **Reynolds transport theorem**

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho\, dV + \int_{CS} b\rho(\vec{V}_r \cdot \vec{n})\, dA$$

$$B = E \qquad b = e \qquad b = e$$

$$\boxed{\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS} e\rho(\vec{V}_r \cdot \vec{n})\, dA}$$

$$e = u + ke + pe = u + V^2/2 + gz \quad \text{(J/kg)}$$

General Energy Equation

The general form of the energy equation that applies to fixed, moving, or deforming control volumes becomes

$$\boxed{\dot{Q}_{net\,in} + \dot{W}_{shaft,net\,in} + \dot{W}_{pressure,net\,in} = \frac{d}{dt}\int_{CV} e\rho d\mathcal{V} + \int_{CS} e\, \underbrace{\rho\left(\vec{V}_r . \vec{n}\right) dA}_{\text{the mass flow rate through area element dA}}}$$

$$\begin{pmatrix}\text{The net rate of energy}\\ \text{transfer into a CV by}\\ \text{heat and work transfer}\end{pmatrix} = \begin{pmatrix}\text{The time rate of}\\ \text{change of the energy}\\ \text{content of the CV}\end{pmatrix} + \begin{pmatrix}\text{The net flow rate of}\\ \text{energy out of the control}\\ \text{surface by mass flow}\end{pmatrix}$$

$$\dot{W}_{pressure,\,net\,in} = -\int_A \frac{P}{\rho}\rho\left(\vec{V}_r . \vec{n}\right) dA \quad \Rightarrow$$

$$\dot{Q}_{net\,in} + \dot{W}_{shaft,net\,in} = \frac{d}{dt}\int_{CV} e\rho d\mathcal{V} + \int_{CS}\left(\frac{P}{\rho} + e\right)\rho\left(\vec{V}_r . \vec{n}\right) dA$$

General Energy Equation

➢The term $P/\rho = w_{flow}$ *is the* ***flow work, which is the work associated*** with pushing a fluid into or out of a control volume per unit mass.

➢The pressure work along the portions of the control surface that coincide with nonmoving solid surfaces is zero.

➢Pressure work for fixed control volumes can exist only along the imaginary part of the control surface where the fluid enters and leaves the control volume, i.e., inlets and outlets.

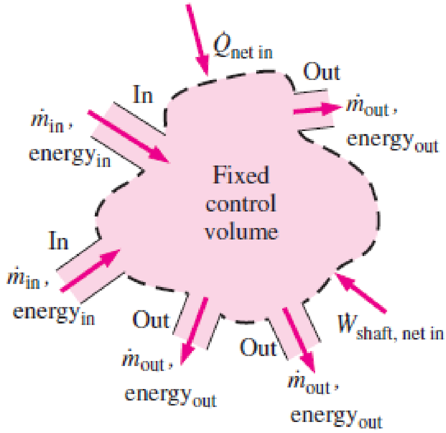
<u>General energy equation for fixed volume :</u>

$$\dot{Q}_{net\,in} + \dot{W}_{shaft,net\,in} = \frac{d}{dt}\int_{CV} e\rho d\mathcal{V} + \int_{CS}\left(\frac{P}{\rho} + e\right)\rho\left(\vec{V} . \vec{n}\right) dA$$

General Energy Equation

$$\dot{Q}_{net\,in}+\dot{W}_{shaft,net\;in}=\frac{d}{dt}\int_{CV}e\rho d\cancel{V}+\int_{CS}\left(\frac{P}{\rho}+e\right)\rho\left(\vec{V}.\vec{n}\right)dA$$

If $P/\rho+e$ *is nearly uniform across an inlet or outlet, we can simply take it* outside the integral.



$$\dot{m}=\int_{A_c}\rho\left(\vec{V}.\vec{n}\right)dA_c$$

$$\dot{Q}_{net\,in}+\dot{W}_{shaft,net\;in}=\frac{d}{dt}\int_{CV}e\rho d\cancel{V}+\sum_{out}\dot{m}\left(\frac{P}{\rho}+e\right)-\sum_{in}\dot{m}\left(\frac{P}{\rho}+e\right)$$

General Energy Equation

$$e=u+ke+pe=u+V^2/2+gz\quad\text{(J/kg)}$$

$$\dot{Q}_{net\,in}+\dot{W}_{shaft,net\;in}=\frac{d}{dt}\int_{CV}e\rho d\cancel{V}+\sum_{out}\dot{m}\left(\frac{P}{\rho}+u+\frac{V^2}{2}+gz\right)-\sum_{in}\dot{m}\left(\frac{P}{\rho}+u+\frac{V^2}{2}+gz\right)$$

Enthalpy : $h=u+\dfrac{P}{\rho}\quad\text{(J/kg)}$

$$\dot{Q}_{net\,in}+\dot{W}_{shaft,net\;in}=\frac{d}{dt}\int_{CV}e\rho d\cancel{V}+\sum_{out}\dot{m}\left(h+\frac{V^2}{2}+gz\right)-\sum_{in}\dot{m}\left(h+\frac{V^2}{2}+gz\right)$$

Above equations are fairly general expressions of conservation of energy

But their use is still limited to fixed control volumes, uniform flow at inlets and outlets, and negligible work due to viscous forces and other effects.

Energy Analysis of Steady Flows

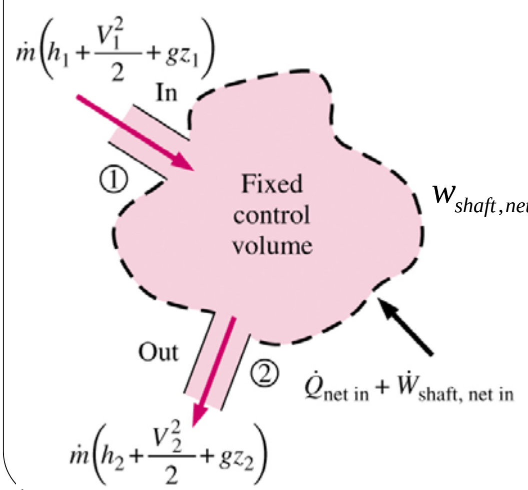
$$Q_{net,in} + W_{shaft,net,in} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

- For steady flow, time rate of change of the energy content of the CV is zero.
- This equation states: *the net rate of energy transfer to a CV by heat and work transfers during steady flow is equal to the difference between the rates of outgoing* and incoming energy flows with mass.

Energy Analysis of Steady Flows

- For **single-stream devices**, mass flow rate is constant.

$$\dot{Q}_{net\ in} + \dot{W}_{shaft,net\ in} = \dot{m}\left(h_2 - h_1 + \frac{V_2^{\ 2} - V_1^{\ 2}}{2} + g(z_2 - z_1)\right)$$



$$q_{net,in} + w_{shaft,net,in} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$w_{shaft,net,in} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{net,in})$$

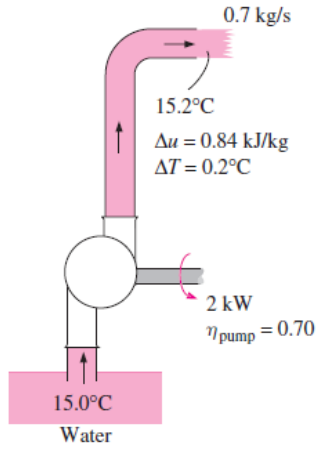
These relations are valid for both compressible and incompressible flows.

Energy Analysis of Steady Flows

$$\underbrace{w_{shaft,in} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1}_{\text{mechanical energy input to the system}} = \underbrace{\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2}_{\text{mechanical energy output from the system}} + \left(u_2 - u_1 - q_{net,in}\right)$$

If the flow is ideal with no irreversibilities such as friction, the total mechanical energy must be conserved, and the term in parentheses (u_2 - u_1 - $q_{net\ in}$) *must equal zero.*

Mechanical energy loss: $e_{mech,loss} = u_2 - u_1 - q_{net,in}$



For single-phase fluids (a gas or a liquid) :

$u_2 - u_1 = c_v(T_2 - T_1)$

c_v *: the constant-volume specific heat.*

Energy Analysis of Steady Flows

The steady-flow energy equation on a unit-mass basis can be written conveniently as a **mechanical energy balance as**

$$e_{mech,in} = e_{mech,out} + e_{mech,loss}$$

$$w_{shaft,net\ in} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{mech,loss}$$

$$w_{shaft,\ \text{net in}} = w_{shaft,\ \text{in}} - w_{shaft,\ \text{out}} = w_{pump} - w_{turbine}$$

The mechanical energy balance can be written more explicitly as

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{turbine} + e_{mech,loss}$$

$e_{mech,loss}$ is the *total* mechanical power loss per unit mass, which consists of pump and turbine losses as well as the frictional losses in the piping network.

Energy Analysis of Steady Flows

By convention, irreversible pump and turbine losses are treated separately from irreversible losses due to other components of the piping system.

In terms of heads: $$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L \quad \text{(m)}$$

$h_{pump,u}$: *useful head delivered to the fluid by the pump*

$h_{turbine,e}$: *extracted head removed from the fluid by the turbine*

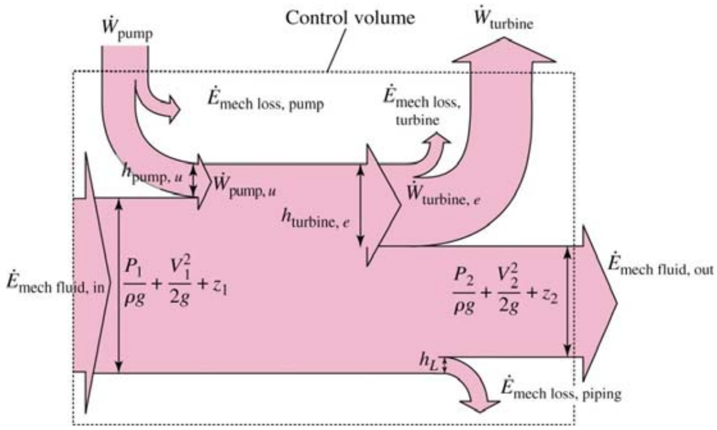
h_L : irreversible *head loss between* 1 and 2 due to all components of the piping system other than the pump or turbine

Energy Analysis of Steady Flows

- Divide by g to get each term in units of length

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine} + h_L$$

Magnitude of each term is now expressed as an equivalent column height of fluid, i.e., *Head*



Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + \cancel{h_{pump}} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + \cancel{h_{turbine}} + \cancel{h_L}$$

No pump

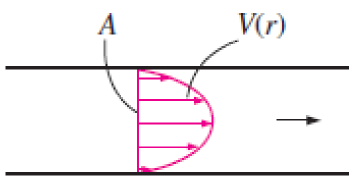
No turbine

Negligible friction

Bernoulli Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad or \quad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

Kinetic Energy Correction Factor, α



for fully developed laminar pipe flow, α = 2

for fully developed turbulent pipe flow, α = 1.04-1.11

$$\dot{m} = \rho V_{avg} A, \qquad \rho = \text{constant}$$

$$\dot{KE}_{act} = \int \text{ke}\,\delta\dot{m} = \int_A \frac{1}{2} V^2(r)[\rho V(r)\,dA]$$

$$= \frac{1}{2}\rho \int_A V^3(r)\,dA$$

$$\dot{KE}_{avg} = \frac{1}{2}\dot{m}V_{avg}^2 = \frac{1}{2}\rho A V_{avg}^3$$

$$\boxed{\alpha = \frac{\dot{KE}_{act}}{\dot{KE}_{avg}} = \frac{1}{A}\int_A \left(\frac{V(r)}{V_{avg}}\right)^3 dA}$$

When the kinetic energy correction factors are included, the energy equations for *steady incompressible flow become*

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L$$

Examples

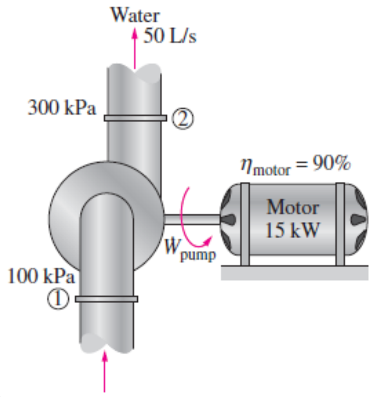
EXAMPLE 5–12 Pumping Power and Frictional Heating in a Pump

The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent (Fig. 5–54). The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine (*a*) the mechanical efficiency of the pump and (*b*) the temperature rise of water as it flows through the pump due to the mechanical inefficiency.

SOLUTION The pressures across a pump are measured. The mechanical efficiency of the pump and the temperature rise of water are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. **3** The elevation difference between the inlet and outlet of the pump is negligible, $z_1 \cong z_2$. **4** The inlet and outlet diameters are the same and thus the inlet and outlet velocities and kinetic energy correction factors are equal, $V_1 = V_2$ and $\alpha_1 = \alpha_2$.

Properties We take the density of water to be 1 kg/L = 1000 kg/m³ and its specific heat to be 4.18 kJ/kg · °C.



Examples

EXAMPLE 5–12 Pumping Power and Frictional Heating in a Pump

Analysis (*a*) The mass flow rate of water through the pump is

$$\dot{m} = \rho\dot{V} = (1 \text{ kg/L})(50 \text{ L/s}) = 50 \text{ kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}}\dot{W}_{\text{electric}} = (0.90)(15 \text{ kW}) = 13.5 \text{ kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

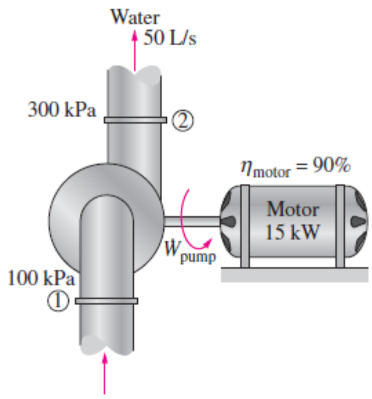
$$\Delta\dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right)$$

Simplifying it for this case and substituting the given values,

$$\Delta\dot{E}_{\text{mech, fluid}} = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = (50 \text{ kg/s})\left(\frac{(300 - 100) \text{ kPa}}{1000 \text{ kg/m}^3}\right)\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right) = 10 \text{ kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta\dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10 \text{ kW}}{13.5 \text{ kW}} = 0.741 \quad \text{or} \quad 74.1\%$$



EXAMPLE 5–12 Pumping Power and Frictional Heating in a Pump

Examples

(*b*) Of the 13.5-kW mechanical power supplied by the pump, only 10 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

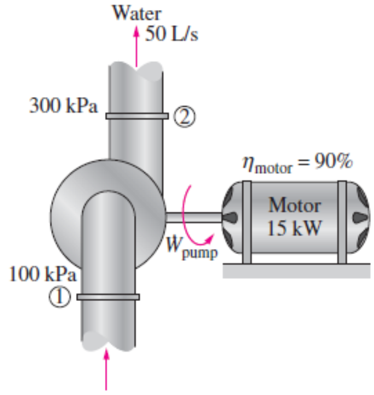
$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta\dot{E}_{\text{mech, fluid}} = 13.5 - 10 = 3.5\text{ kW}$$

The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance, $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$. Solving for ΔT,

$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}c} = \frac{3.5\text{ kW}}{(50\text{ kg/s})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C})} = 0.017^\circ\text{C}$$

Therefore, the water will experience a temperature rise of 0.017°C due to mechanical inefficiency, which is very small, as it flows through the pump.

Discussion In an actual application, the temperature rise of water will probably be less since part of the heat generated will be transferred to the casing of the pump and from the casing to the surrounding air. If the entire pump motor were submerged in water, then the 1.5 kW dissipated to the air due to motor inefficiency would also be transferred to the surrounding water as heat. This would cause the water temperature to rise more.



EXAMPLE 5–13 Hydroelectric Power Generation from a Dam

In a hydroelectric power plant, 100 m³/s of water flows from an elevation of 120 m to a turbine, where electric power is generated (Fig. 5–55). The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output.

SOLUTION The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electric power output is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be 1000 kg/m³.

Analysis The mass flow rate of water through the turbine is

$$\dot{m} = \rho\dot{V} = (1000\text{ kg/m}^3)(100\text{ m}^3/\text{s}) = 10^5\text{ kg/s}$$

We take point 2 as the reference level, and thus $z_2 = 0$. Also, both points 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{\text{atm}}$) and the flow velocities are negligible at both points ($V_1 = V_2 = 0$). Then the energy equation for steady, incompressible flow reduces to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \cancelto{0}{h_{\text{pump}, u}} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cancelto{0}{z_2} + h_{\text{turbine}, e} + h_L \rightarrow$$

$$h_{\text{turbine}, e} = z_1 - h_L$$

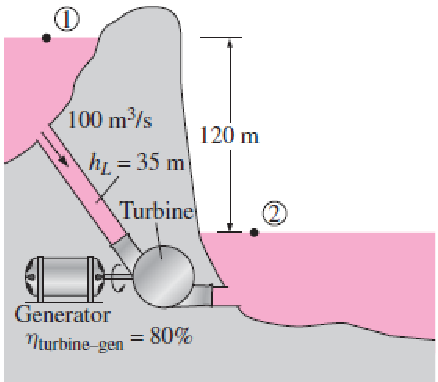
Substituting, the extracted turbine head and the corresponding turbine power are

$$h_{\text{turbine}, e} = z_1 - h_L = 120 - 35 = 85\text{ m}$$

$$\dot{W}_{\text{turbine}, e} = \dot{m}gh_{\text{turbine}, e} = (10^5\text{ kg/s})(9.81\text{ m/s}^2)(85\text{ m})\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 83{,}400\text{ kW}$$

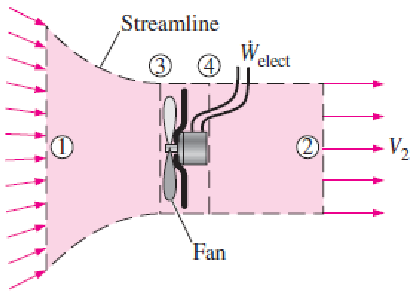
Therefore, a perfect turbine–generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{\text{electric}} = \eta_{\text{turbine–gen}}\dot{W}_{\text{turbine}, e} = (0.80)(83.4\text{ MW}) = 66.7\text{ MW}$$



EXAMPLE 5–14 **Fan Selection for Air Cooling of a Computer**

A fan is to be selected to cool a computer case whose dimensions are 12 cm × 40 cm × 40 cm (Fig. 5–56). Half of the volume in the case is expected to be filled with components and the other half to be air space. A 5-cm-diameter hole is available at the back of the case for the installation of the fan that is to replace the air in the void spaces of the case once every second. Small low-power fan–motor combined units are available in the market and their efficiency is estimated to be 30 percent. Determine (*a*) the wattage of the fan–motor unit to be purchased and (*b*) the pressure difference across the fan. Take the air density to be 1.20 kg/m³.



SOLUTION A fan is to cool a computer case by completely replacing the air inside once every second. The power of the fan and the pressure difference across it are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Losses other than those due to the inefficiency of the fan–motor unit are negligible ($h_L = 0$). **3** The flow at the outlet is fairly uniform except near the center (due to the wake of the fan motor), and the kinetic energy correction factor at the outlet is 1.10.

Properties The density of air is given to be 1.20 kg/m³.

Analysis (*a*) Noting that half of the volume of the case is occupied by the components, the air volume in the computer case is

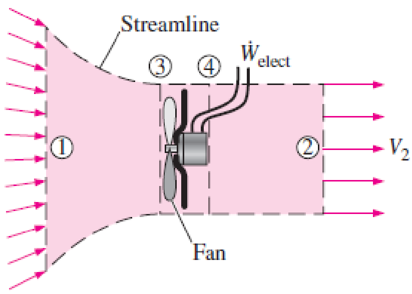
$$V = \text{(Void fraction)(Total case volume)}$$
$$= 0.5(12 \text{ cm} \times 40 \text{ cm} \times 40 \text{ cm}) = 9600 \text{ cm}^3$$

$$\dot{V} = \frac{V}{\Delta t} = \frac{9600 \text{ cm}^3}{1 \text{ s}} = 9600 \text{ cm}^3/\text{s} = 9.6 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\dot{m} = \rho\dot{V} = (1.20 \text{ kg/m}^3)(9.6 \times 10^{-3} \text{ m}^3/\text{s}) = 0.0115 \text{ kg/s}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.05 \text{ m})^2}{4} = 1.96 \times 10^{-3} \text{ m}^2$$

$$V = \frac{\dot{V}}{A} = \frac{9.6 \times 10^{-3} \text{ m}^3/\text{s}}{1.96 \times 10^{-3} \text{ m}^2} = 4.90 \text{ m/s}$$



We draw the control volume around the fan such that both the inlet and the outlet are at atmospheric pressure ($P_1 = P_2 = P_{atm}$), as shown in Fig. 5–56, and the inlet section 1 is large and far from the fan so that the flow velocity at the inlet section is negligible ($V_1 \cong 0$). Noting that $z_1 = z_2$ and frictional losses in flow are disregarded, the mechanical losses consist of fan losses only and the energy equation (Eq. 5–76) simplifies to

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1 \cancelto{0}{\frac{V_1^2}{2}} + \cancel{gz_1}\right) + \dot{W}_{fan} = \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2 \frac{V_2^2}{2} + \cancel{gz_2}\right) + \cancelto{0}{\dot{W}_{turbine}} + \dot{E}_{mech\ loss,\ fan}$$

Solving for $\dot{W}_{fan} - \dot{E}_{mech\ loss,\ fan} = \dot{W}_{fan,\ u}$ and substituting,

$$\dot{W}_{fan,\ u} = \dot{m}\alpha_2 \frac{V_2^2}{2} = (0.0115 \text{ kg/s})(1.10)\frac{(4.90 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 0.152 \text{ W}$$

Then the required electric power input to the fan is determined to be

$$\dot{W}_{elect} = \frac{\dot{W}_{fan,\ u}}{\eta_{fan-motor}} = \frac{0.152 \text{ W}}{0.3} = \mathbf{0.506 \text{ W}}$$

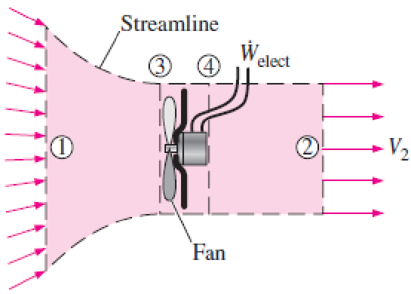
(*b*) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. This time again $z_3 = z_4$ and $V_3 = V_4$ since the fan is a narrow cross section, and the energy equation reduces to

$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{\text{fan}} = \dot{m}\frac{P_4}{\rho} + \dot{E}_{\text{mech loss, fan}} \quad \rightarrow \quad \dot{W}_{\text{fan, }u} = \dot{m}\frac{P_4 - P_3}{\rho}$$

Solving for $P_4 - P_3$ and substituting,

$$P_4 - P_3 = \frac{\rho \dot{W}_{\text{fan, }u}}{\dot{m}} = \frac{(1.2\ \text{kg/m}^3)(0.152\ \text{W})}{0.0115\ \text{kg/s}}\left(\frac{1\ \text{Pa}\cdot\text{m}^3}{1\ \text{Ws}}\right) = \mathbf{15.8\ Pa}$$

Discussion The efficiency of the fan–motor unit is given to be 30 percent, which means 30 percent of the electric power $\dot{W}_{\text{electric}}$ consumed by the unit is converted to useful mechanical energy while the rest (70 percent) is "lost" and converted to thermal energy. Also, a more powerful fan is required in an actual system to overcome frictional losses inside the computer case. Note that if we had ignored the kinetic energy correction factor at the outlet, the required electrical power and pressure rise would have been 10 percent lower in this case (0.460 W and 14.4 Pa, respectively).



EXAMPLE 5–15 Head and Power Loss During Water Pumping

Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to the water (Fig. 5–57). The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m³/s, determine the irreversible head loss of the system and the lost mechanical power during this process.

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1 \frac{V_1^2}{2}^{\nearrow 0} + gz_1^{\nearrow 0}\right) + \dot{W}_{\text{pump}}$$

$$= \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2 \frac{V_2^2}{2}^{\nearrow 0} + gz_2\right) + \dot{W}_{\text{turbine}}^{\nearrow 0} + \dot{E}_{\text{mech, loss}}$$

$$\dot{W}_{\text{pump}} = \dot{m}gz_2 + \dot{E}_{\text{mech, loss}} \quad \rightarrow \quad \dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump}} - \dot{m}gz_2$$

Substituting, the lost mechanical power and head loss are determined to be

$$\dot{E}_{\text{mech, loss}} = 20\ \text{kW} - (30\ \text{kg/s})(9.81\ \text{m/s}^2)(45\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)$$

$$= \mathbf{6.76\ kW}$$

Noting that the entire mechanical losses are due to frictional losses in piping and thus $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech, loss, piping}}$, the irreversible head loss is determined to be

$$h_L = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g} = \frac{6.76\ \text{kW}}{(30\ \text{kg/s})(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)\left(\frac{1000\ \text{N}\cdot\text{m/s}}{1\ \text{kW}}\right) = \mathbf{23.0\ m}$$