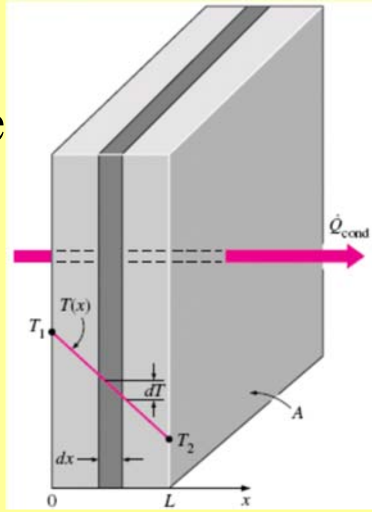
Chapter 3: Steady Heat Conduction

Objectives

When you finish studying this chapter, you should be able to:

- Understand the concept of thermal resistance and its limitations, and develop thermal resistance networks for practical heat conduction problems,
- Solve steady conduction problems that involve multilayer rectangular, cylindrical, or spherical geometries,
- Develop an intuitive understanding of thermal contact resistance, and circumstances under which it may be significant,
- Identify applications in which insulation may actually increase heat transfer,
- Analyze finned surfaces, and assess how efficiently and effectively fins enhance heat transfer, and
- Solve multidimensional practical heat conduction problems using conduction shape factors.

Steady Heat Conduction in Plane Walls



1) Considerable temperature difference between the inner and the outer surfaces of the wall (significant temperature gradient in the x direction).

2) The wall surface is nearly *isothermal*.

Steady one-dimensional modeling approach is justified.

- Assuming heat transfer is the only energy interaction and there is no heat generation, the *energy balance* can be expressed as

Zero for steady operation

$$\left(\begin{array}{c}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{array}\right) - \left(\begin{array}{c}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{array}\right) = \overbrace{\left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{array}\right)}^{\text{Zero for steady operation}}$$

or

$$\dot{Q}_{in} - \dot{Q}_{out} = \overbrace{\frac{dE_{wall}}{dt}}^{=0} = 0 \qquad \textbf{(3-1)}$$

The rate of heat transfer through the wall must be constant ($\dot{Q}_{cond,wall} = \text{constant}$).

- Then Fourier's law of heat conduction for the wall can be expressed as

$$\dot{Q}_{cond,wall} = -kA\frac{dT}{dx} \qquad \text{(W)} \qquad \textbf{(3-2)}$$

- Remembering that the rate of conduction heat transfer and the wall area *A* are constant it follows

$\Rightarrow$ dT/dx=constant

$\Downarrow$

the temperature through the wall varies linearly with x.

- Integrating the above equation and rearranging yields

$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)} \qquad \textbf{(3-3)}$$

Thermal Resistance Concept- **Conduction** Resistance

- Equation 3–3 for heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{R_{wall}} \qquad \text{(W)} \qquad \textbf{(3-4)}$$

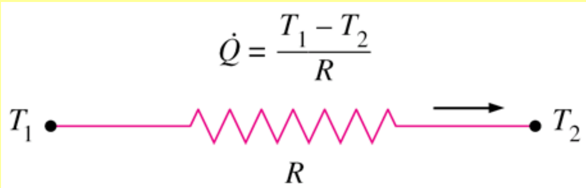
- Where R_{wall} is the **conduction resistance** expressed as

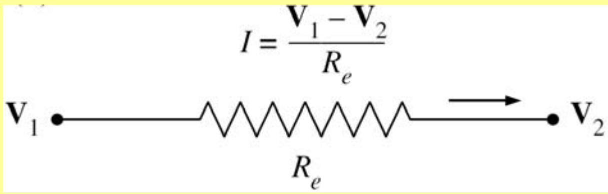
$$R_{wall} = \frac{L}{kA} \qquad (^\circ\text{C/W}) \qquad \textbf{(3-5)}$$

Analogy to Electrical Current Flow

- Eq. 3-5 is analogous to the relation for *electric current flow* *I*, expressed as

$$I = \frac{V_1 - V_2}{R_e} \qquad \textbf{(3-6)}$$





Heat Transfer		Electrical current flow
Rate of heat transfer	←→	*Electric current*
Thermal resistance	←→	*Electrical resistance*
Temperature difference	←→	*Voltage difference*

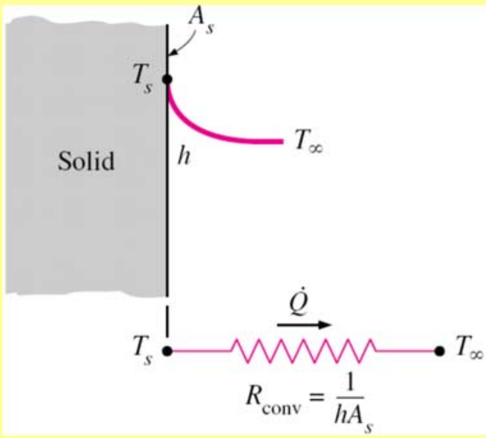
Thermal Resistance Concept- **Convection** Resistance

- Thermal resistance can also be applied to convection processes.
- Newton's law of cooling for convection heat transfer rate ($\dot{Q}_{conv} = hA_s(T_s - T_\infty)$) can be rearranged as

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \qquad \text{(W)} \qquad \textbf{(3-7)}$$

- R_{conv} is the convection resistance

$$R_{conv} = \frac{1}{hA_s} \qquad (^\circ\text{C/W}) \qquad \textbf{(3-8)}$$



Thermal Resistance Concept-
Radiation Resistance

- The rate of radiation heat transfer between a surface and the surrounding

$$\dot{Q}_{rad} = \varepsilon\sigma A_s\left(T_s^4 - T_{surr}^4\right) = h_{rad} A_s (T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \text{(W)}$$

(3-9)

$$R_{rad} = \frac{1}{h_{rad} A_s} \quad (K/\text{W})$$ **(3-10)**

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right) \ (\text{W/m}^2 \cdot \text{K})$$

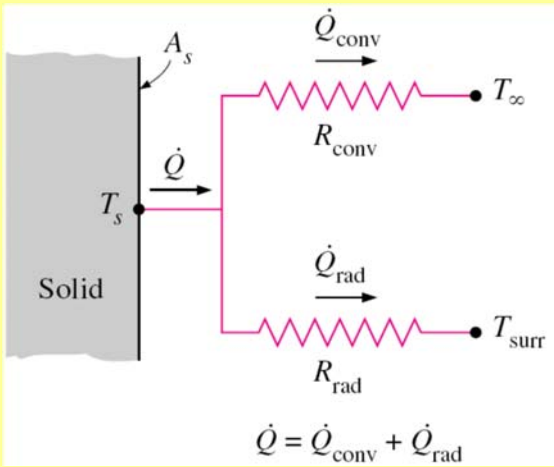
(3-11)

Thermal Resistance Concept-
Radiation and **Convection** Resistance

- A surface exposed to the surrounding might involves convection and radiation simultaneously.
- The convection and radiation resistances are parallel to each other.
- When $T_{surr} \approx T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation by

$$h_{combined} = h_{conv} + h_{rad} \text{(W/m}^2\text{K)}$$

(3-12)



Thermal Resistance Network

- consider steady one-dimensional heat transfer through a plane wall that is exposed to convection on both sides.
- Under steady conditions we have

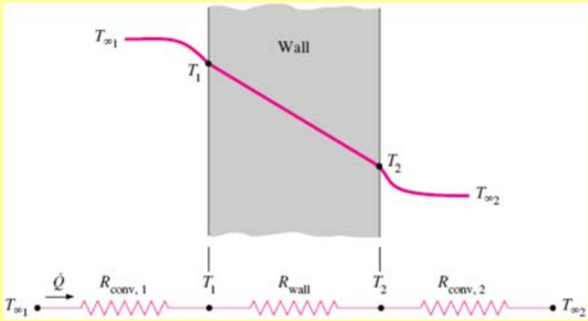
$$\left(\begin{array}{c}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{array}\right) = \left(\begin{array}{c}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{array}\right) = \left(\begin{array}{c}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{array}\right)$$

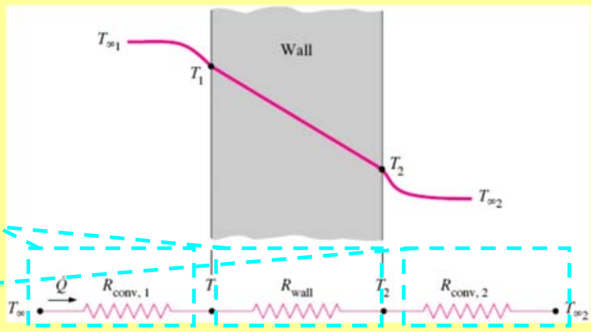
or

$$\dot{Q} = h_1 A\left(T_{\infty,1} - T_1\right) =$$

$$kA\frac{T_1 - T_2}{L} = h_2 A\left(T_2 - T_{\infty,2}\right)$$

(3-13)





Rearranging and adding

$$+\begin{cases} T_{\infty,1} - \cancel{T_1} = \dot{Q}\cdot R_{conv,1} \\ \cancel{T_1} - \cancel{T_2} = \dot{Q}\cdot R_{wall} \\ \cancel{T_2} - T_{\infty,2} = \dot{Q}\cdot R_{conv,2} \end{cases}$$

$$\boxed{T_{\infty,1} - T_{\infty,2}} = Q(R_{conv,1} + R_{wall} + R_{conv,2}) = Q\cdot R_{total}$$

$$\dot{Q} = \frac{T_{\infty,1} - T_{\infty,2}}{R_{total}} \quad \text{(W)}$$ **(3-15)**

where

$$R_{total} = R_{conv,1} + R_{wall} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (^\circ\text{C/W})$$

(3-16)

- It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA\Delta T \qquad (\mathrm{W}) \qquad \textbf{(3-18)}$$

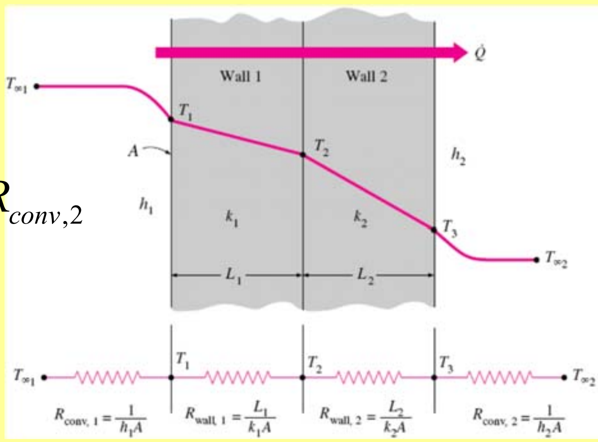
- where U is the **overall heat transfer coefficient**.
- Note that

$$UA = \frac{1}{R_{total}} \qquad (^{\circ}\mathrm{C/K}) \qquad \textbf{(3-19)}$$

Multilayer Plane Walls

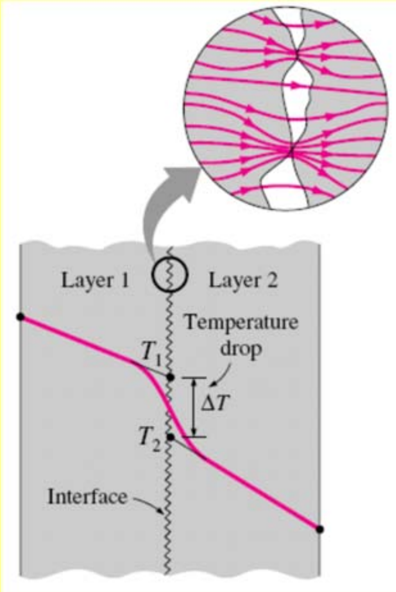
- In practice we often encounter plane walls that consist of several layers of different materials.
- The rate of steady heat transfer through this two-layer composite wall can be expressed through Eq. 3-15 where the total thermal resistance is

$$\begin{aligned} R_{total} &= R_{conv,1} + R_{wall,1} + R_{wall,2} + R_{conv,2} \\ &= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \end{aligned} \qquad \textbf{(3-22)}$$



Thermal Contact Resistance

- In reality surfaces have some roughness.
- When two surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air.
- As a result, an interface contains numerous *air gaps* of varying sizes that act as *insulation* because of the low thermal conductivity of air.
- Thus, an **interface offers** some **resistance to heat transfer**, which is termed the **thermal contact resistance**, R_c.



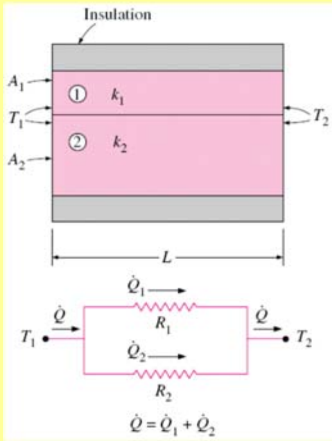
- The value of thermal contact resistance depends on the
  - surface roughness,
  - material properties,
  - temperature and pressure at the interface,
  - type of fluid trapped at the interface.
- Thermal contact resistance is observed to decrease with decreasing surface roughness and increasing interface pressure.
- The thermal contact resistance can be minimized by applying a thermally conducting liquid called a thermal grease.

Generalized Thermal Resistance Network

- The *thermal resistance* concept can be used to solve steady heat transfer problems that involve parallel layers or combined series-parallel arrangements.
- The total heat transfer of two parallel layers

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}_{\boxed{\frac{1}{R_{total}}}} \quad \textbf{(3-29)}$$

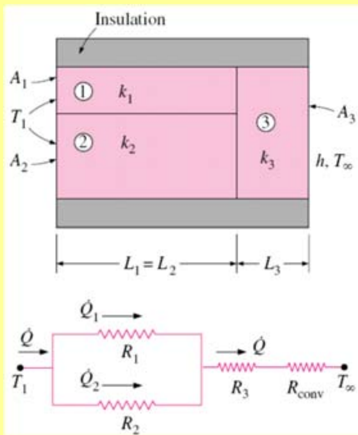
$$\frac{1}{R_{total}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \rightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2} \quad \textbf{(3-31)}$$



Combined Series-Parallel Arrangement

The total rate of heat transfer through the composite system

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \quad \textbf{(3-32)}$$



where

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv} \quad \textbf{(3-33)}$$

$$R_1 = \frac{L_1}{k_1 A_1} \; ; R_2 = \frac{L_2}{k_2 A_2} \; ; R_3 = \frac{L_3}{k_3 A_3} \; ; R_{conv} = \frac{1}{h A_3} \quad \textbf{(3-34)}$$

Heat Conduction in Cylinders

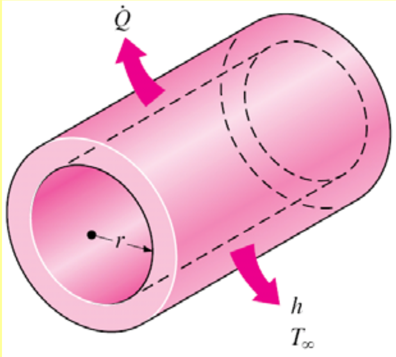
Consider the long cylindrical layer

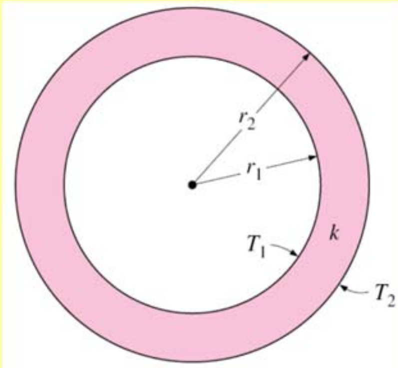
Assumptions:

- the two surfaces of the cylindrical layer are maintained at constant temperatures T_1 and T_2,
- no heat generation,
- constant thermal conductivity,
- one-dimensional heat conduction.

Fourier's law of heat conduction

$$\dot{Q}_{cond,cyl} = -kA\frac{dT}{dr} \quad \text{(W)} \qquad \textbf{(3-35)}$$





$$\dot{Q}_{cond,cyl} = -kA\frac{dT}{dr} \quad \text{(W)} \qquad \textbf{(3-35)}$$

Separating the variables and integrating from $r=r_1$, where $T(r_1)=T_1$, to $r=r_2$, where $T(r_2)=T_2$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{cond,cyl}}{A}dr = -\int_{T=T_1}^{T_2} kdT \qquad \textbf{(3-36)}$$

Substituting $A = 2\pi rL$ and performing the integrations give

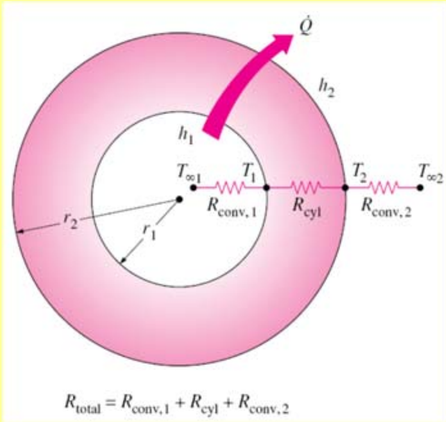
$$\dot{Q}_{cond,cyl} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \textbf{(3-37)}$$

Since the heat transfer rate is constant

$$\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{R_{cyl}} \qquad \textbf{(3-38)}$$

Thermal Resistance with Convection

Steady one-dimensional heat transfer through a cylindrical or spherical layer that is exposed to convection on both sides



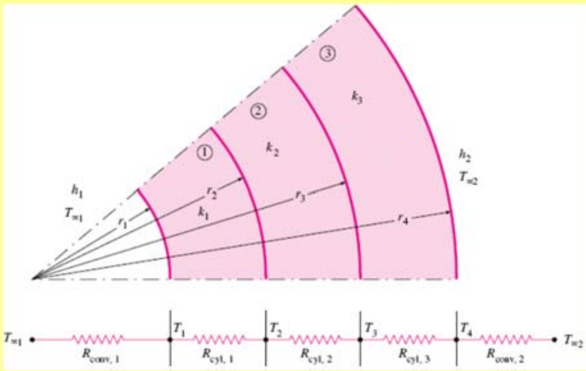
$$\dot{Q} = \frac{T_{\infty,1} - T_{\infty,2}}{R_{total}} \qquad \textbf{(3-32)}$$

where

$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2} =$$

$$= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{(2\pi r_2 L)h_2} \qquad \textbf{(3-43)}$$

Multilayered Cylinders



- Steady heat transfer through multilayered cylindrical or spherical shells can be handled just like multilayered plane.
- The steady heat transfer rate through a three-layered composite cylinder of length L with convection on both sides is expressed by Eq. 3-32 where:

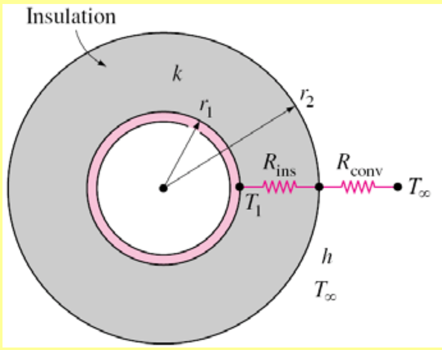
$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,3} + R_{cyl,3} + R_{conv,2} = \qquad \textbf{(3-46)}$$

$$= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk_1} + \frac{\ln(r_3/r_2)}{2\pi Lk_2} + \frac{\ln(r_4/r_3)}{2\pi Lk_3} + \frac{1}{(2\pi r_2 L)h_2}$$

Critical Radius of Insulation

- Adding more insulation to a wall or to the attic always decreases heat transfer.
- Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter.
- Adding insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.
- The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

- A cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant.
- The pipe is covered with an insulator (k and r_2).
- Convection heat transfer at T_∞ and h.



- The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as

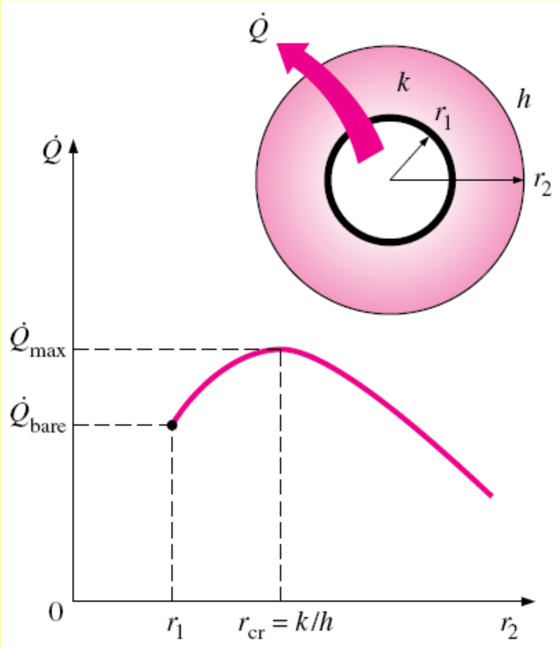
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}} \qquad \textbf{(3-37)}$$

- The variation of the heat transfer rate with the outer radius of the insulation r_2 is shown in the figure.
- The value of r_2 at which $\dot{Q}$ reaches a maximum is determined by

$$\frac{d\dot{Q}}{dr_2} = 0$$

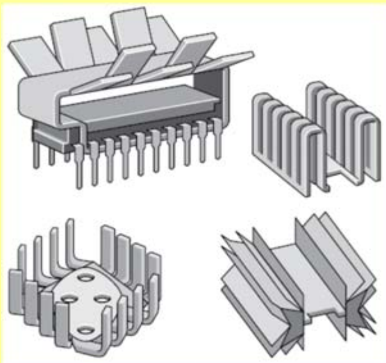
- Performing the differentiation and solving for r_2 yields

$$r_{cr,cylinder} = \frac{k}{h} \quad \text{(m)} \quad \textbf{(3-50)}$$



- Thus, insulating the pipe may actually increase the rate of heat transfer instead of decreasing it.

Heat Transfer from Finned Surfaces



- Newton's law of cooling

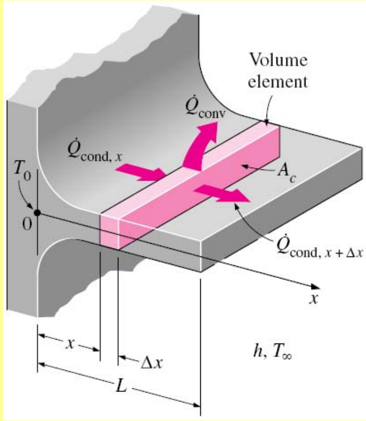
$$\dot{Q}_{conv} = hA_s\left(T_s - T_\infty\right)$$

- Two ways to increase the rate of heat transfer:
  - increasing the heat transfer coefficient,
  - increase the surface area → fins
- Fins are the topic of this section.

Fin Equation

Under steady conditions, the energy balance on this volume element can be expressed as

$$\left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction into}\\ \text{the element at } x\end{array}\right) = \left(\begin{array}{c}\text{Rate of } \textit{heat}\\ \textit{conduction} \text{ from the}\\ \text{element at } x+\Delta x\end{array}\right) + \left(\begin{array}{c}\text{Rate of heat}\\ \text{convection from}\\ \text{the element}\end{array}\right)$$



or $$\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv}$$

where $$\dot{Q}_{conv} = h(p\Delta x)(T - T_\infty)$$

Substituting and dividing by Δx, we obtain

$$\frac{\dot{Q}_{cond,x+\Delta x} - \dot{Q}_{cond,x}}{\Delta x} + hp(T - T_\infty) = 0 \qquad \textbf{(3-52)}$$

Taking the limit as $\Delta x \rightarrow 0$ gives

$$\frac{d\dot{Q}_{cond}}{dx} + hp(T - T_\infty) = 0 \qquad \textbf{(3-53)}$$

From Fourier's law of heat conduction we have

$$\dot{Q}_{cond} = -kA_c \frac{dT}{dx} \qquad \textbf{(3-54)}$$

Substitution of Eq. 3-54 into Eq. 3–53 gives

$$\frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hp(T - T_\infty) = 0 \qquad \textbf{(3-55)}$$

For constant cross section and constant thermal conductivity

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad \textbf{(3-56)}$$

Where

$$\theta = T - T_\infty \;\; ; \;\; m = \frac{hp}{kA_c}$$

- Equation 3–56 is a linear, homogeneous, second-order differential equation with constant coefficients.
- The general solution of Eq. 3–56 is

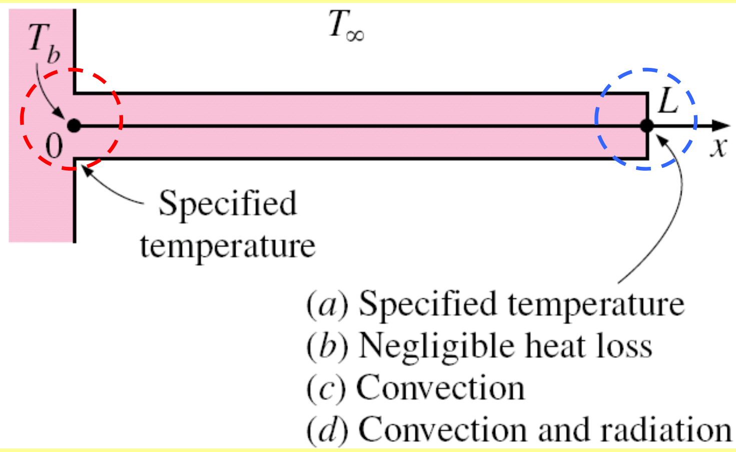
$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad \textbf{(3-58)}$$

- C_1 and C_2 are constants whose values are to be determined from the boundary conditions at the base and at the tip of the fin.

Boundary Conditions

Several boundary conditions are typically employed:

- At the fin base
  - **Specified temperature** boundary condition, expressed as: $\theta(0) = \theta_b = T_b - T_\infty$
- At the fin tip
  1. Specified temperature
  2. Infinitely Long Fin
  3. Adiabatic tip
  4. Convection (and combined convection and radiation).



Infinitely Long Fin ($T_{\text{fin tip}}=T$)

- For a sufficiently long fin the temperature at the fin tip approaches the ambient temperature

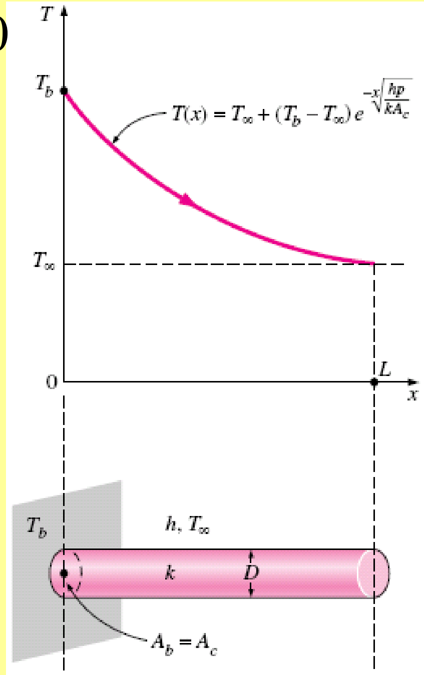
Boundary condition: $\theta(L\to\infty)=T(L)-T_\infty=0$

- When $x\to\infty$ so does $e^{mx}\to\infty$ ⇒ $C_1=0$
- @ $x=0$: $e^{mx}=1$ ⇒ $C_2=\theta_b$
- The temperature distribution:

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=e^{-mx}=e^{-x\sqrt{hp/kA_c}} \qquad \textbf{(3-60)}$$

- *heat transfer* from the entire fin

$$\dot{Q}=-kA_c\left.\frac{dT}{dx}\right|_{x=0}=\sqrt{hpkA_c}\left(T_b-T_\infty\right) \qquad \textbf{(3-61)}$$



Adiabatic Tip

- Boundary condition at fin tip:

$$\left.\frac{d\theta}{dx}\right|_{x=L}=0 \qquad \textbf{(3-63)}$$

- After some manipulations, the temperature distribution:

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m\left(L-x\right)}{\cosh mL} \qquad \textbf{(3-64)}$$

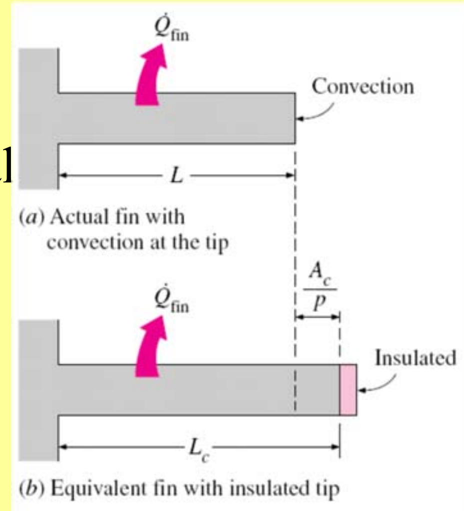
- *heat transfer* from the entire fin

$$\dot{Q}=-kA_c\left.\frac{dT}{dx}\right|_{x=0}=\sqrt{hpkA_c}\left(T_b-T_\infty\right)\tanh mL \qquad \textbf{(3-65)}$$

Convection (or Combined Convection and Radiation) from Fin Tip

- A practical way of accounting for the heat loss from the fin tip is to replace the *fin length L* in the relation for the *insulated tip* case by a **corrected length** defined as

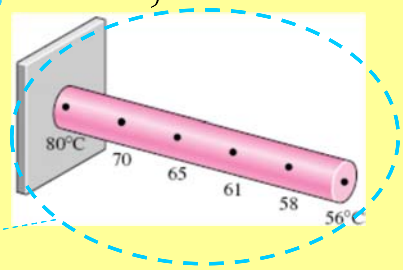
$$L_c = L + A_c/p \quad \textbf{(3-66)}$$



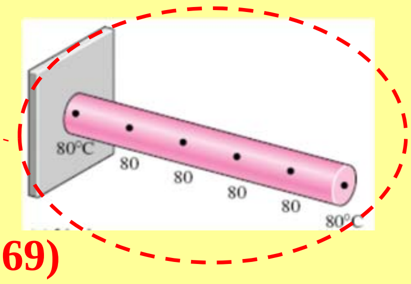
- For rectangular and cylindrical fins L_c is
- $L_{c,rectangular} = L + t/2$
- $L_{c,cylindrical} = L + D/4$

Fin Efficiency

- To maximize the heat transfer from a fin the temperature of the fin should be uniform (maximized) at the base value of T_b
- In reality, the temperature drops along the fin, and thus the heat transfer from the fin is less
- To account for the effect we define a **fin efficiency**



$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$



or

$$\dot{Q}_{fin} = \eta_{fin}\dot{Q}_{fin,\max} = \eta_{fin} h A_{fin}(T_b - T_\infty) \quad \textbf{(3-69)}$$

Fin Efficiency

- For constant cross section of very long fins:

$$\eta_{long,fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\sqrt{hpkA_c}\left(T_b - T_\infty\right)}{hA_{fin}\left(T_b - T_\infty\right)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL} \quad \textbf{(3-70)}$$

- For constant cross section with adiabatic tip:

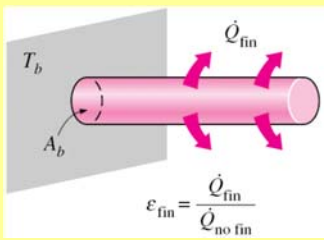
$$\eta_{adiabatic,fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\sqrt{hpkA_c}\left(T_b - T_\infty\right)\tanh aL}{hA_{fin}\left(T_b - T_\infty\right)} = \frac{\tanh mL}{mL} \quad \textbf{(3-71)}$$

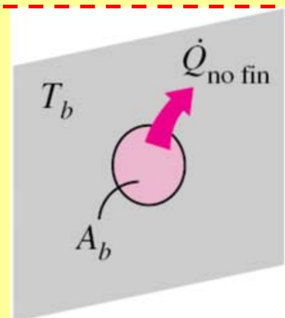
Fin Effectiveness

- The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case.
- The performance of fins is expressed in terms of the *fin effectiveness* ε_{fin} defined as

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\dot{Q}_{fin}}{hA_b\left(T_b - T_\infty\right)} = \frac{\text{Heat transfer rate from the fin of } base\ area\ A_b}{\text{Heat transfer rate from the surface of } area\ A_b}$$

(3-72)





Remarks regarding fin effectiveness

- The *thermal conductivity* k of the fin material should be as high as possible. It is no coincidence that fins are made from metals.
- The ratio of the *perimeter* to the *cross-sectional area* of the fin p/A_c should be as high as possible.
- The use of fins is *most effective* in applications involving a *low convection heat transfer coefficient.*

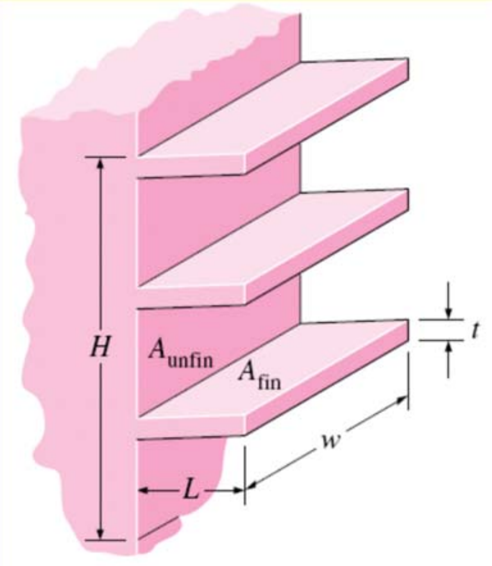
The use of fins is more easily justified when the medium is a *gas* instead of a liquid and the heat transfer is by *natural convection* instead of by forced convection.

Overall Effectiveness

- An **overall effectiveness** for a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins.

$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} \quad \textbf{(3-76)}$$

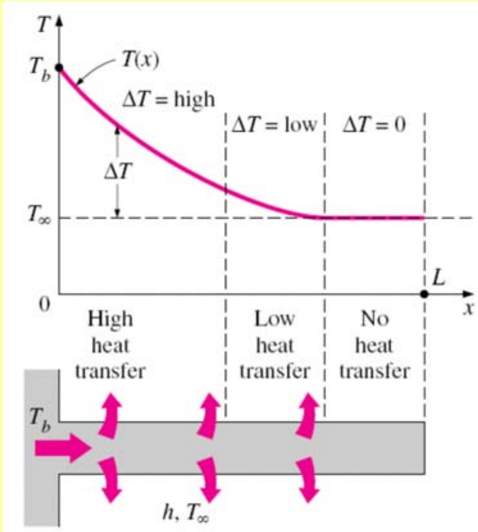
$$= \frac{h\left(A_{unfin} + \eta_{fin} A_{fin}\right)}{h A_{no\ fin}}$$



$$A_{\text{no fin}} = w \times H$$
$$A_{\text{unfin}} = w \times H - 3 \times (t \times w)$$
$$A_{\text{fin}} = 2 \times L \times w + t \times w$$
$$\cong 2 \times L \times w \text{ (one fin)}$$

Proper Length of a Fin

- An important step in the design of a fin is the determination of the appropriate length of the fin once the fin material and the fin cross section are specified.
- The temperature drops along the fin exponentially and asymptotically approaches the ambient temperature at some length.



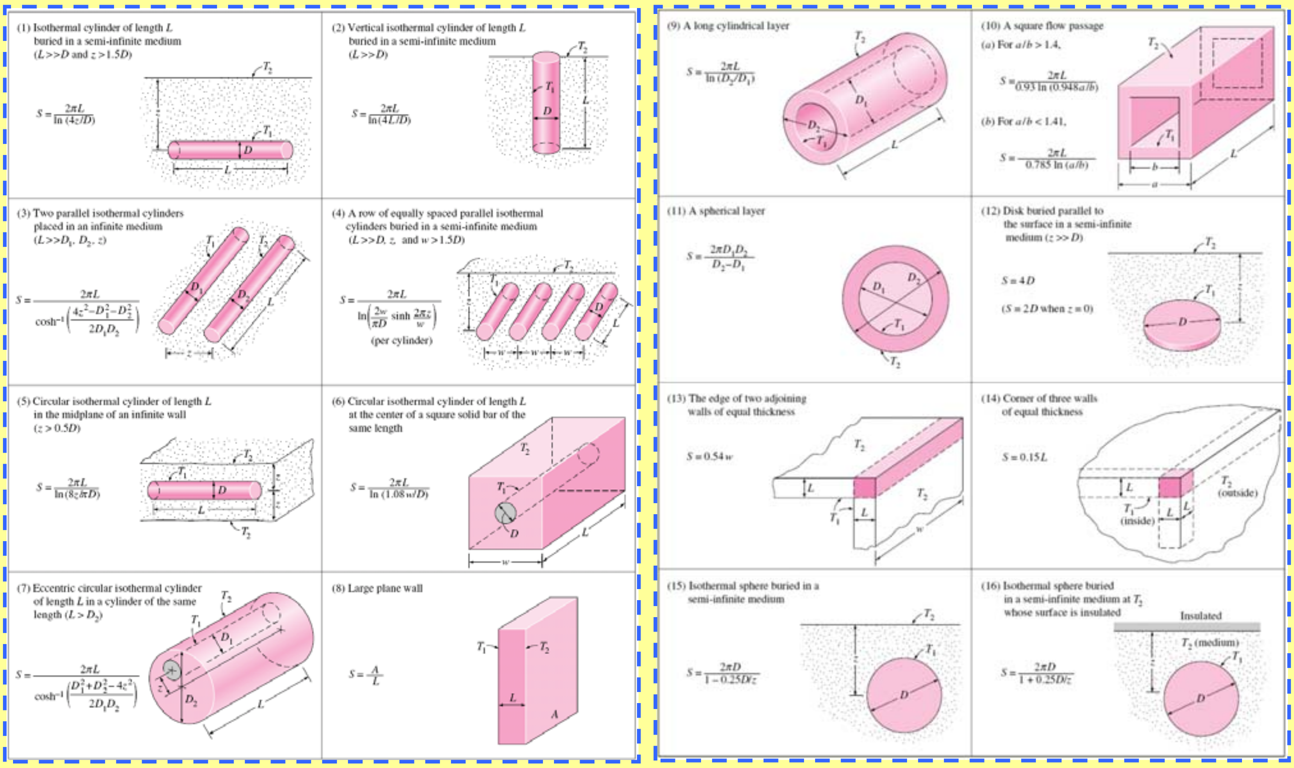
Heat Transfer in Common Configurations

- Many problems encountered in practice are two- or three-dimensional and involve rather complicated geometries for which no simple solutions are available.
- An important class of heat transfer problems for which simple solutions are obtained encompasses those involving two surfaces maintained at *constant* temperatures T_1 and T_2.
- The steady rate of heat transfer between these two surfaces is expressed as

$$Q=Sk(T_1-T_2) \qquad \textbf{(3-79)}$$

- S is the **conduction shape factor**, which has the dimension of *length*.

Table 3-7



(1) Isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$ and $z > 1.5D$)	$S = \dfrac{2\pi L}{\ln(4z/D)}$
(2) Vertical isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$)	$S = \dfrac{2\pi L}{\ln(4L/D)}$
(3) Two parallel isothermal cylinders placed in an infinite medium ($L >> D_1, D_2, z$)	$S = \dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$
(4) A row of equally spaced parallel isothermal cylinders buried in a semi-infinite medium ($L >> D, z,$ and $w > 1.5D$)	$S = \dfrac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\sinh\dfrac{2\pi z}{w}\right)}$ (per cylinder)
(5) Circular isothermal cylinder of length L in the midplane of an infinite wall ($z > 0.5D$)	$S = \dfrac{2\pi L}{\ln(8z/\pi D)}$
(6) Circular isothermal cylinder of length L at the center of a square solid bar of the same length	$S = \dfrac{2\pi L}{\ln(1.08\,w/D)}$
(7) Eccentric circular isothermal cylinder of length L in a cylinder of the same length ($L > D_2$)	$S = \dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4z^2}{2D_1D_2}\right)}$
(8) Large plane wall	$S = \dfrac{A}{L}$
(9) A long cylindrical layer	$S = \dfrac{2\pi L}{\ln(D_2/D_1)}$
(10) A square flow passage	(a) For $a/b > 1.4$, $S = \dfrac{2\pi L}{0.93\ln(0.948a/b)}$; (b) For $a/b < 1.41$, $S = \dfrac{2\pi L}{0.785\ln(a/b)}$
(11) A spherical layer	$S = \dfrac{2\pi D_1D_2}{D_2 - D_1}$
(12) Disk buried parallel to the surface in a semi-infinite medium ($z >> D$)	$S = 4D$ ($S = 2D$ when $z = 0$)
(13) The edge of two adjoining walls of equal thickness	$S = 0.54w$
(14) Corner of three walls of equal thickness	$S = 0.15L$
(15) Isothermal sphere buried in a semi-infinite medium	$S = \dfrac{2\pi D}{1 - 0.25D/z}$
(16) Isothermal sphere buried in a semi-infinite medium at T_2 whose surface is insulated	$S = \dfrac{2\pi D}{1 + 0.25D/z}$