2–57 Consider a large plane wall of thickness $L = 0.3$ m, thermal conductivity $k = 2.5$ W/m·K, and surface area $A = 12$ m^2. The left side of the wall at $x = 0$ is subjected to a net heat flux of $\dot{q}_0 = 700$ W/m^2 while the temperature at that surface is measured to be $T_1 = 80°C$. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the temperature of the right surface of the wall at $x = L$. *Answer: (c) −4°C*

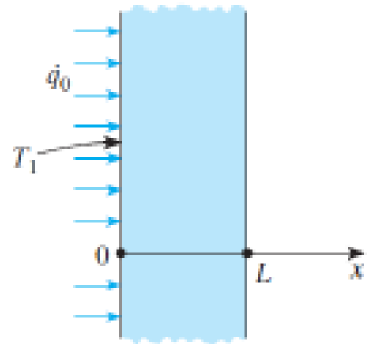


FIGURE P2–57

$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 700 \text{ W/m}^2$$

$$T(0) = T_1 = 80°\text{C}$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions

Heat flux at $x = 0$: $$-kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

Temperature at $x = 0$: $$T(0) = C_1 \times 0 + C_2 = T_1 \rightarrow C_2 = T_1$$

Substituting C_1 and C_2 into the general solution, the variation of temperature

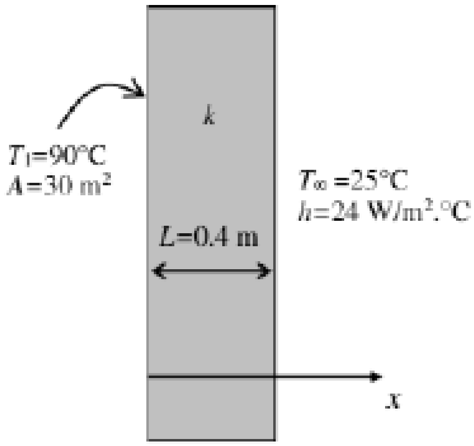
$$T(x) = -\frac{\dot{q}_0}{k}x + T_1 = -\frac{700 \text{ W/m}^2}{2.5 \text{ W/m}\cdot°\text{C}}x + 80°\text{C} = -280x + 80$$

(*c*) The temperature at $x = L$ (the right surface of the wall) is

$$T(L) = -280 \times (0.3 \text{ m}) + 80 = \mathbf{-4°C}$$

Note that the right surface temperature is lower as expected.

2–59 Consider a large plane wall of thickness $L = 0.4$ m, thermal conductivity $k = 1.8$ W/m·K, and surface area $A = 30\ \text{m}^2$. The left side of the wall is maintained at a constant temperature of $T_1 = 90°\text{C}$ while the right side loses heat by convection to the surrounding air at $T_\infty = 25°\text{C}$ with a heat transfer coefficient of $h = 24\ \text{W/m}^2\cdot\text{K}$. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the rate of heat transfer through the wall. *Answer: (c) 7389 W*



$$\frac{d^2T}{dx^2} = 0$$

and

$$T(0) = T_1 = 90°\text{C}$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$x = 0$: $\qquad T(0) = C_1 \times 0 + C_2 \ \rightarrow\ C_2 = T_1$

$x = L$: $\qquad -kC_1 = h[(C_1 L + C_2) - T_\infty] \rightarrow\ C_1 = -\dfrac{h(C_2 - T_\infty)}{k + hL} \rightarrow\ C_1 = -\dfrac{h(T_1 - T_\infty)}{k + hL}$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= -\frac{h(T_1 - T_\infty)}{k + hL}x + T_1 \\ &= -\frac{(24\ \text{W/m}^2\cdot°\text{C})(90 - 25)°\text{C}}{(1.8\ \text{W/m}\cdot°\text{C}) + (24\ \text{W/m}^2\cdot°\text{C})(0.4\ \text{m})}x + 90°\text{C} \\ &= 90 - 90.3x \end{aligned}$$

(*c*) The rate of heat conduction through the wall is

$$\begin{aligned} \dot{Q}_{\text{wall}} &= -kA\frac{dT}{dx} = -kAC_1 = kA\frac{h(T_1 - T_\infty)}{k + hL} \\ &= (1.8\ \text{W/m}\cdot°\text{C})(30\ \text{m}^2)\frac{(24\ \text{W/m}^2\cdot°\text{C})(90 - 25)\ °\text{C}}{(1.8\ \text{W/m}\cdot°\text{C}) + (24\ \text{W/m}^2\cdot°\text{C})(0.4\ \text{m})} \\ &= \mathbf{7389\ W} \end{aligned}$$

2–148 When a long section of a compressed air line passes through the outdoors, it is observed that the moisture in the compressed air freezes in cold weather, disrupting and even completely blocking the air flow in the pipe. To avoid this problem, the outer surface of the pipe is wrapped with electric strip heaters and then insulated. Consider a compressed air pipe of length $L = 6$ m, inner radius $r_1 = 3.7$ cm, outer radius $r_2 = 4.0$ cm, and thermal conductivity $k = 14$ W/m·K equipped with a 300-W strip heater. Air is flowing through the pipe at an average temperature of $-10°C$, and the average convection heat transfer coefficient on the inner surface is $h = 30$ W/m²·K. Assuming 15 percent of the heat generated in the strip heater is lost through the insulation, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the pipe, (*b*) obtain a relation for the variation of temperature in the pipe material by solving the differential equation, and (*c*) evaluate the inner and outer surface temperatures of the pipe.

$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{2\pi r_2 L} = \frac{0.85 \times 300 \text{ W}}{2\pi(0.04 \text{ m})(6 \text{ m})} = 169.1 \text{ W/m}^2$$

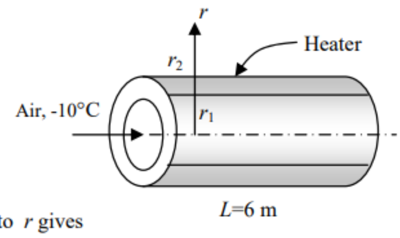
Noting that heat transfer is one-dimensional in the radial r direction and heat flux is in the negative r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and

$$-k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$



(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $\quad k\frac{C_1}{r_2} = \dot{q}_s \;\rightarrow\; C_1 = \frac{\dot{q}_s r_2}{k}$

$r = r_1$: $\quad -k\frac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)] \rightarrow C_2 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln r - \ln r_1 + \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln\frac{r}{r_1} + \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$$

$$= -10°\text{C} + \left(\ln\frac{r}{r_1} + \frac{14 \text{ W/m}\cdot°\text{C}}{(30 \text{ W/m}^2\cdot°\text{C})(0.037 \text{ m})}\right)\frac{(169.1 \text{ W/m}^2)(0.04 \text{ m})}{14 \text{ W/m}\cdot°\text{C}} = -10 + 0.483\left(\ln\frac{r}{r_1} + 12.61\right)$$

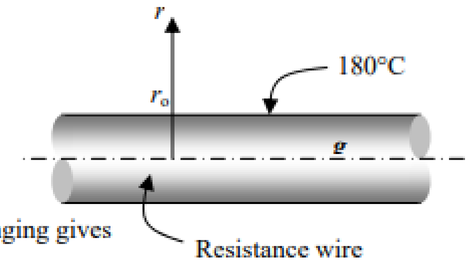
(*c*) The inner and outer surface temperatures are determined by direct substitution to be

Inner surface ($r = r_1$): $T(r_1) = -10 + 0.483\left(\ln\frac{r_1}{r_1} + 12.61\right) = -10 + 0.483(0 + 12.61) = \mathbf{-3.91°C}$

Outer surface ($r = r_2$): $T(r_1) = -10 + 0.483\left(\ln\frac{r_2}{r_1} + 12.61\right) = -10 + 0.483\left(\ln\frac{0.04}{0.037} + 12.61\right) = \mathbf{-3.87°C}$

Note that the pipe is essentially isothermal at a temperature of about -3.9°C.

2–99 A long homogeneous resistance wire of radius r_o = 5 mm is being used to heat the air in a room by the passage of electric current. Heat is generated in the wire uniformly at a rate of 5×10^7 W/m^3 as a result of resistance heating. If the temperature of the outer surface of the wire remains at 180°C, determine the temperature at r = 3.5 mm after steady operation conditions are reached. Take the thermal conductivity of the wire to be k = 6 W/m·K. *Answer:* 207°C



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0$$

and $\quad T(r_0)=T_s=180°\text{C}$ (specified surface temperature)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and rearranging gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{g}}{k}r$$

Integrating with respect to r gives

$$r\frac{dT}{dr}=-\frac{\dot{g}}{k}\frac{r^2}{2}+C_1 \qquad (a)$$

It is convenient at this point to apply the boundary condition at the center since it is related to the first derivative of the temperature. It yields

B.C. at $r=0$: $\quad 0\times\frac{dT(0)}{dr}=-\frac{\dot{g}}{2k}\times 0+C_1 \quad \rightarrow \quad C_1=0$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{g}}{2k}r$$

and $$T(r)=-\frac{\dot{g}}{4k}r^2+C_2 \qquad (b)$$

Applying the other boundary condition at $r=r_0$,

B. C. at $r=r_0$: $\quad T_s=-\frac{\dot{g}}{4k}r_0^2+C_2 \quad \rightarrow \quad C_2=T_s+\frac{\dot{g}}{4k}r_0^2$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r)=T_s+\frac{\dot{g}}{4k}(r_0^2-r^2)$$

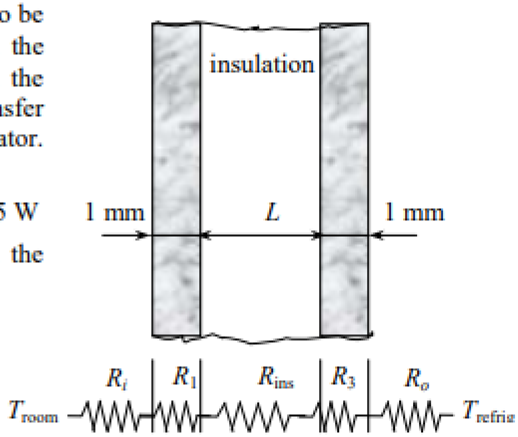
which is the desired solution for the temperature distribution in the wire as a function of r. The temperature 2 mm from the center line (r = 0.002 m) is determined by substituting the known quantities to be

$$T(0.002\text{ m})=T_s+\frac{\dot{g}}{4k}(r_0^2-r^2)=180°\text{C}+\frac{5\times10^7\ \text{W/m}^3}{4\times(8\ \text{W/ m.°C})}[(0.005\text{ m})^2-(0.002\text{ m})^2]=\mathbf{212.8°C}$$

Thus the temperature at that location will be about 33°C above the temperature of the outer surface of the wire.

3–35 The wall of a refrigerator is constructed of fiberglass insulation ($k = 0.035$ W/m·K) sandwiched between two layers of 1-mm-thick sheet metal ($k = 15.1$ W/m·K). The refrigerated space is maintained at 3°C, and the average heat transfer coefficients at the inner and outer surfaces of the wall are 4 W/m^2·K and 9 W/m^2·K, respectively. The kitchen temperature averages 25°C. It is observed that condensation occurs on the outer surfaces of the refrigerator when the temperature of the outer surface drops to 20°C. Determine the minimum thickness of fiberglass insulation that needs to be used in the wall in order to avoid condensation on the outer surfaces.

Analysis The minimum thickness of insulation can be determined by assuming the outer surface temperature of the refrigerator to be 10°C. In steady operation, the rate of heat transfer through the refrigerator wall is constant, and thus heat transfer between the room and the refrigerated space is equal to the heat transfer between the room and the outer surface of the refrigerator. Considering a unit surface area,



$$\dot{Q} = h_o A(T_{room} - T_{s,out}) = (9\ \text{W/m}^2.°\text{C})(1\ \text{m}^2)(25-20)°\text{C} = 45\ \text{W}$$

Using the thermal resistance network, heat transfer between the room and the refrigerated space can be expressed as

$$\dot{Q} = \frac{T_{room} - T_{refrig}}{R_{total}}$$

$$\dot{Q}/A = \frac{T_{room} - T_{refrig}}{\dfrac{1}{h_o} + 2\left(\dfrac{L}{k}\right)_{metal} + \left(\dfrac{L}{k}\right)_{insulation} + \dfrac{1}{h_i}}$$

Substituting,

$$45\ \text{W/m}^2 = \frac{(25-3)°\text{C}}{\dfrac{1}{9\ \text{W/m}^2.°\text{C}} + \dfrac{2 \times 0.001\ \text{m}}{15.1\ \text{W/m}^2.°\text{C}} + \dfrac{L}{0.035\ \text{W/m}^2.°\text{C}} + \dfrac{1}{4\ \text{W/m}^2.°\text{C}}}$$

Solv ing for L, the minimum thickness of insulation is determined to be

$L = 0.0045\ \text{m} =$ **0.45 cm**

To defrost ice accumulated on the outer surface of an automobile windshield, warm air is blown over the inner surface of the windshield. Consider an automobile windshield with thickness of 5 mm and thermal conductivity of 1.4 W/m·K. The outside ambient temperature is −10°C and the convection heat transfer coefficient is 200 W/m2·K, while the ambient temperature inside the automobile is 25°C. Determine the value of the convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield necessary to cause the accumulated ice to begin melting.

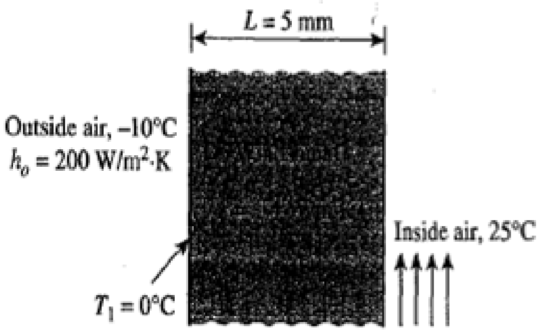


FIGURE P3-41

Given data:

Thermal conductivity of the wind shield, $k = 1.4\,\text{W/m-K}$.

Thickness of the wind shield, $L = 5\,\text{mm} = 0.005\,\text{m}$.

Temperature of outside air, $T_{\infty,o} = -10\,°\text{C}$.

Temperature of inside air, $T_{\infty,i} = 25\,°\text{C}$.

Heat transfer coefficient at the outer surface of the wind shield, $h_o = 200\,\text{W/m}^2\text{-K}$.

Temperature of the outer surface the wind shield, $T_1 = 0\,°\text{C}$.

Step 2 of 5

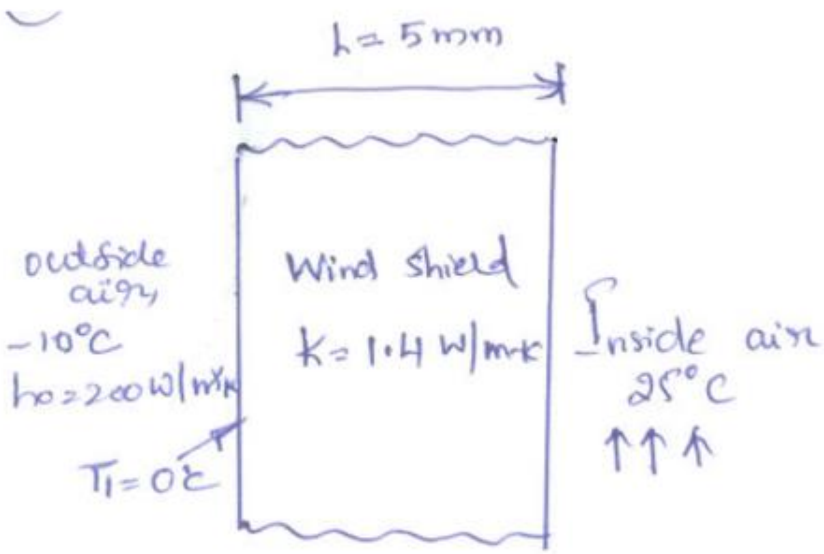


Figure 1 Schematic

The temperature of the outer surface of the wind shield assumed to be $T_1 = 0\,^\circ\text{C}$, since the accumulated ice to begin melting. In order to determine the convective heat transfer coefficient for the warm air blowing over inner surface of the wind shield, the heat flux through the wind shield to be determined. Therefore, the heat flux through the wind shield can be calculated as

$$\begin{aligned}\dot{q} &= h_o(T_1 - T_{\infty,o}) \\ &= 200\,(\text{W/m}^2\text{-K}) \times \left(0-(-10)\right) \\ &= 2000\ \text{W/m}^2\end{aligned}$$

Step 4 of 5

By using the thermal resistance network approach, the total thermal resistance can be calculated as

$$\dot{q} = \frac{\Delta T}{R_{total}}$$
$$\rightarrow R_{total} = \frac{\Delta T}{\dot{q}}$$

$$\begin{aligned}R_{total} &= \frac{T_{\infty,i} - T_{\infty,o}}{\dot{q}} \\ &= \frac{(25-(-10))}{2000(\text{W/m}^2)} \\ &= 0.0175\,\text{m}^2\text{-K/W}\end{aligned}$$

The total thermal resistance per unit area can be expressed as

$$R_{total} = \frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}$$

By re writing the above equation, the convective heat transfer coefficient for the warm air blowing over inner surface of the wind shield can be expressed as

$$\begin{aligned}h_i &= \frac{1}{\left(R_{total} - \left(\frac{1}{h_o} + \frac{L}{k}\right)\right)} \\ &= \frac{1}{\left(0.0175(\text{m}^2\text{-K/W}) - \left(\frac{1}{200(\text{W/m}^2\text{-K})} + \frac{0.005(\text{K})}{1.4(\text{W/m-K})}\right)\right)} \\ &= 112\,\text{W/m}^2\text{-K}\end{aligned}$$

$\boxed{h_i = 112\,\text{W/m}^2\text{-K}}$ Therefore, convective heat transfer coefficient is