2–57 Consider a large plane wall of thickness $L = 0.3$ m, thermal conductivity $k = 2.5$ W/m·K, and surface area $A = 12$ m^2. The left side of the wall at $x = 0$ is subjected to a net heat flux of $\dot{q}_0 = 700$ W/m^2 while the temperature at that surface is measured to be $T_1 = 80°$C. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the temperature of the right surface of the wall at $x = L$. *Answer: (c)* $-4°$C

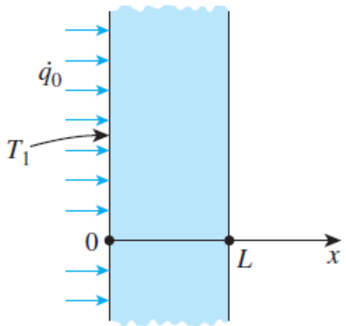


FIGURE P2–57

$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 700 \text{ W/m}^2$$

$$T(0) = T_1 = 80°\text{C}$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions

Heat flux at $x = 0$: $\quad -kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$

Temperature at $x = 0$: $\quad T(0) = C_1 \times 0 + C_2 = T_1 \rightarrow C_2 = T_1$

Substituting C_1 and C_2 into the general solution, the variation of temperature

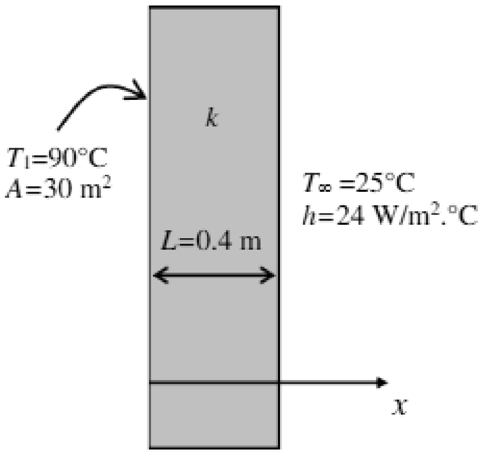
$$T(x) = -\frac{\dot{q}_0}{k}x + T_1 = -\frac{700 \text{ W/m}^2}{2.5 \text{ W/m}\cdot°\text{C}}x + 80°\text{C} = -280x + 80$$

(*c*) The temperature at $x = L$ (the right surface of the wall) is

$$T(L) = -280 \times (0.3 \text{ m}) + 80 = \mathbf{-4°C}$$

Note that the right surface temperature is lower as expected.

2–59 Consider a large plane wall of thickness L = 0.4 m, thermal conductivity k = 1.8 W/m·K, and surface area A = 30 m^2. The left side of the wall is maintained at a constant temperature of T_1 = 90°C while the right side loses heat by convection to the surrounding air at T_∞ = 25°C with a heat transfer coefficient of h = 24 W/m^2·K. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the rate of heat transfer through the wall. *Answer:* (*c*) 7389 W



$$\frac{d^2T}{dx^2}=0$$

and

$$T(0)=T_1=90°\text{C}$$

$$-k\frac{dT(L)}{dx}=h[T(L)-T_\infty]$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx}=C_1$$

$$T(x)=C_1x+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$x=0$: $\quad T(0)=C_1\times 0+C_2 \;\rightarrow\; C_2=T_1$

$x=L$: $\quad -kC_1=h[(C_1L+C_2)-T_\infty] \rightarrow C_1=-\dfrac{h(C_2-T_\infty)}{k+hL} \rightarrow C_1=-\dfrac{h(T_1-T_\infty)}{k+hL}$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\begin{aligned}T(x)&=-\frac{h(T_1-T_\infty)}{k+hL}x+T_1\\&=-\frac{(24\text{ W/m}^2\cdot°\text{C})(90-25)°\text{C}}{(1.8\text{ W/m}\cdot°\text{C})+(24\text{ W/m}^2\cdot°\text{C})(0.4\text{ m})}x+90°\text{C}\\&=90-90.3x\end{aligned}$$

(*c*) The rate of heat conduction through the wall is

$$\begin{aligned}\dot{Q}_{\text{wall}}&=-kA\frac{dT}{dx}=-kAC_1=kA\frac{h(T_1-T_\infty)}{k+hL}\\&=(1.8\text{ W/m}\cdot°\text{C})(30\text{ m}^2)\frac{(24\text{ W/m}^2\cdot°\text{C})(90-25)°\text{C}}{(1.8\text{ W/m}\cdot°\text{C})+(24\text{ W/m}^2\cdot°\text{C})(0.4\text{ m})}\\&=\mathbf{7389\text{ W}}\end{aligned}$$

2–69 A pipe in a manufacturing plant is transporting superheated vapor at a mass flow rate of 0.3 kg/s. The pipe is 10 m long, has an inner diameter of 5 cm and pipe wall thickness of 6 mm. The pipe has a thermal conductivity of 17 W/m·K, and the inner pipe surface is at a uniform temperature of 120°C. The temperature drop between the inlet and exit of the pipe is 7°C, and the constant pressure specific heat of vapor is 2190 J/kg·°C. If the air temperature in the manufacturing plant is 25°C, determine the heat transfer coefficient as a result of convection between the outer pipe surface and the surrounding air.

Analysis The inner and outer radii of the pipe are

$$r_1 = 0.05\,\text{m}/2 = 0.025\,\text{m}$$

$$r_2 = 0.025\,\text{m} + 0.006\,\text{m} = 0.031\,\text{m}$$

The rate of heat loss from the vapor in the pipe can be determined from

$$\dot{Q}_{\text{loss}} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = (0.3\,\text{kg/s})(2190\,\text{J/kg}\cdot{}^\circ\text{C})(7)\,{}^\circ\text{C} = 4599\,\text{W}$$

For steady one-dimensional heat conduction in cylindrical coordinates, the heat conduction ec

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and $$-k\frac{dT(r_1)}{dr} = \frac{\dot{Q}_{\text{loss}}}{A} = \frac{\dot{Q}_{\text{loss}}}{2\pi r_1 L} \quad \text{(heat flux at the inner pipe surface)}$$

$$T(r_1) = 120\,{}^\circ\text{C} \quad \text{(inner pipe surface temperature)}$$

Integrating the differential equation once with respect to r gives

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Integrating with respect to r again gives

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions gives

$$r = r_1: \qquad \frac{dT(r_1)}{dr} = -\frac{1}{k}\frac{\dot{Q}_{\text{loss}}}{2\pi r_1 L} = \frac{C_1}{r_1} \quad \rightarrow \quad C_1 = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}$$

$$r = r_1: \qquad T(r_1) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + C_2 \quad \rightarrow \quad C_2 = \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + T(r_1)$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined t

$$T(r) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r + \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + T(r_1)$$
$$= -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r/r_1) + T(r_1)$$

The outer pipe surface temperature is

$$T(r_2) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r_2/r_1) + T(r_1)$$
$$= -\frac{1}{2\pi}\frac{4599\,\text{W}}{(17\,\text{W/m}\cdot{}^\circ\text{C})(10\,\text{m})}\ln\left(\frac{0.031}{0.025}\right) + 120\,{}^\circ\text{C}$$
$$= 119.1\,{}^\circ\text{C}$$

From Newton's law of cooling, the rate of heat loss at the outer pipe surface by convectic

$$\dot{Q}_{\text{loss}} = h(2\pi r_2 L)[T(r_2) - T_\infty]$$

Rearranging and the convection heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}_{\text{loss}}}{2\pi r_2 L[T(r_2) - T_\infty]} = \frac{4599\,\text{W}}{2\pi(0.031\,\text{m})(10\,\text{m})(119.1 - 25)\,{}^\circ\text{C}} = \mathbf{25.1\ W/m^2\cdot{}^\circ C}$$

Chapter 2, Problem 146P

Problem

Exhaust gases from a manufacturing plant are being discharged through a 10-m tall exhaust Stack with outer diameter of 1 m, wall thickness of 10 cm, and thermal conductivity of 40 W/m · K. The exhaust gases are discharged at a rate of 1.2 kg/s, while temperature drop between inlet and exit of the exhaust Stack is 30 °C, and the constant pressure specific heat of the exhaust gasses is 1600 J/kg · K. On a particular day, the outer surface of the exhaust Stack experiences radiation with the surrounding at 27 °C, and convection with the ambient air at 27 °C also, with an average convection heat transfer coefficient of 8 W/m2 · K. Solar radiation is incident on the exhaust Stack outer surface at a rate of 150 W/m2, and both the emissivity and solar absorptivity of the outer surface are 0.9. Assuming steady one-dimensional heat transfer, (*a*) obtain the Variation of temperature in the exhaust Stack wall and (*b*) determine the inner surface temperature of the exhaust Stack.

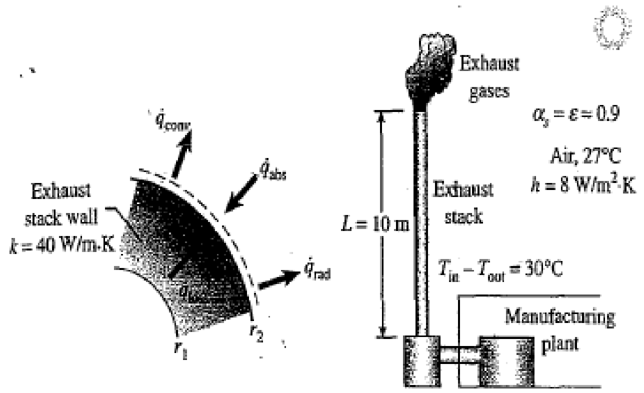


FIGURE P2–151

Fundamentals of Engineering (FE) Exam Problems

Calculate the outer radius of the exhaust stack.

$$r_2 = \frac{d_2}{2}$$

Here, d_2 is the outer diameter of the exhaust stack.

Substitute 1 m for d_2.

$$\begin{aligned} r_2 &= \frac{1}{2} \\ &= 0.5 \text{ m} \end{aligned}$$

Calculate the inner radius of the stack,

$$r_1 = r_2 - t$$

Here, *t* is the wall thickness of the stack.

Substitute 0.5 m for r_2, and 0.1 m for *t*.

$$\begin{aligned} r_1 &= 0.5 - 0.1 \\ &= 0.4 \text{ m} \end{aligned}$$

Step 2 of 7

Calculate the surface area of the exhaust stack by using the relation,

$$A_s = 2\pi r_2 L$$

Here, *L* is the length of the exhaust stack.

Substitute 0.5 m for r_2, and 10 m for *L*.

$$\begin{aligned} A_s &= 2\pi \times 0.5 \times 10 \\ &= 31.416 \text{ m}^2 \end{aligned}$$

Calculate the heat carried out by the exhaust gases by using the relation,

$$Q_{\text{lost}} = mc_p \Delta T_{\text{gas}}$$

Here, *m* is the mass flow rate of the exhaust gases, c_p is the specific heat of exhaust gas at constant pressure, and ΔT_{gas} is the temperature drop between the inlet and exit of the exhaust stack.

Substitute 1.2 kg/s for *m*, $1600 \text{ J/kg} \cdot {}^\circ\text{C}$ for c_p, and 30°C for ΔT_{gas}.

$$\begin{aligned} Q_{\text{lost}} &= 1.2 \times 1600 \times 30 \\ &= 57600 \text{ W} \end{aligned}$$

Step 3 of 7

Calculate the outer surface temperature of the pipe by using the relation,

$$\frac{Q_{\text{lost}}}{A_s} = h\left(T_{r_2} - T_\infty\right) + \varepsilon\sigma\left(T_{r_2}^4 - T_\infty^4\right) - \alpha_s q_{solar}$$

Here, *h* is the heat transfer coefficient of air, T_{r_2} is the outer surface temperature of the pipe, T_∞ is the temperature of the surrounding air, ε is the emissivity of the outer surface, σ is the Stephen Boltzmann constant, α_s is the absorptivity of the outer surface, and q_{solar} is the rate of heat transfer by radiation at the outer surface of the pipe.

Substitute 57600 W for Q_{lost}, 31.416 m^2 for A_s, $8 \text{ W/m}^2\cdot\text{K}$ for *h*, 300 K for T_∞, 0.9 for α_s, and 150 W/m^2 for q_{solar}.

$$\frac{57600}{31.416} = 8\left(T_{r_2} - 300\right) + 0.9\times5.67\times10^{-8}\left(T_{r_2}^4 - 300^4\right) - 0.9\times150$$
$$1833.46 = 8T_{r_2} - 2400 + 5.103\times10^{-8}T_{r_2}^4 - 413.343 - 135$$
$$5.103\times10^{-8}T_{r_2}^4 + 8T_{r_2} = 1833.46 + 2400 + 413.343 + 135$$
$$5.103\times10^{-8}T_{r_2}^4 + 8T_{r_2} = 4781.803$$
$$T_{r_2} = 412.694 \text{ K}$$

Step 4 of 7

Write the equation for the heat flux at the inner surface of the stack.

$$\frac{Q_{lost}}{A_s} = -k\frac{dT_{r1}}{dr}$$
$$= \frac{Q_{lost}}{2\pi r_1 L}$$
$$\frac{dT_{r1}}{dr} = -\frac{Q_{lost}}{2\pi k r_1 L}$$

Write the one dimensional heat conduction equation for the cylindrical coordinates,

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrate with respect to *r*.

$$\int_0^r \frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$
$$r\frac{dT}{dr} = C_1 \qquad \text{...... (1)}$$
$$\frac{dT}{dr} = \frac{C_1}{r}$$

Apply the boundary condition, $r = r_1$.

$$\frac{dT_{r1}}{dr} = \frac{C_1}{r_1}$$
$$= -\frac{Q_{lost}}{2\pi k r_1 L}$$
$$C_1 = -\frac{Q_{lost}}{2\pi k L}$$

Again integrate the equation (1) with respect to *r*.

$T_r = C_1 \ln r + C_2$ ...... (2)

Here, C_1 and C_2 are the arbitrary constants.

Apply the boundary condition, $r = r_2$.

$$T_{r2} = -\frac{Q_{lost}}{2\pi k L}\ln r_2 + C_2$$
$$C_2 = T_{r2} + \frac{Q_{lost}}{2\pi k L}\ln r_2$$

Step 6 of 7

Substitute the values of C_1 and C_2 in equation (2)

$$T_r = \left(-\frac{Q_{lost}}{2\pi k L}\right)\ln r + \left(T_{r2} + \frac{Q_{lost}}{2\pi k L}\ln r_2\right)$$
$$= -\frac{Q_{lost}}{2\pi k L}\ln r + T_{r2} + \frac{Q_{lost}}{2\pi k L}\ln r_2$$
$$= -\frac{Q_{lost}}{2\pi k L}\ln\left(r/r_2\right) + T_{r2}$$

Therefore, the relation for the variation of temperature in the exhaust stack is

$$\boxed{-\frac{Q_{lost}}{2\pi k L}\ln\left(r/r_2\right) + T_{r2}}$$

Step 7 of 7

b)

Calculate the inner surface temperature of the exhaust stack by using the relation,

$$T_{r1} = -\frac{Q_{lost}}{2\pi k L}\ln\left(r_1/r_2\right) + T_{r2}$$

Substitute 57600 W for Q_{lost}, $40\ \text{W/m}\cdot\text{K}$ for *k*, 10 m for *L*, 0.4 m for r_1, 0.5 m for r_2, and $412.694\ \text{K}$ for T_{r2}.

$$T_{r1} = -\frac{57600}{2\pi \times 40 \times 10}\ln\left(0.4/0.5\right) + 412.694$$
$$= 417.7\ \text{K}$$

Therefore, the inner surface temperature of the exhaust stack is $\boxed{417.7\ \text{K}}$.

Chapter 2, Problem 143P

Problem

A 1200-W ironIs left on the iron board with its base exposed to ambient air at 26°C. The base plate of the iron has a thickness of L = 0.5 cm, base area of A = 150 cm2, and thermal conductivity of k = 18 W/m · K. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside. The outer surface of the base plate whose emissivity is ε = 0.7, loses heat by convection to ambient air with an average heat transfer coefficient of h = 30 W/m2 · K as well as by radiation to the surrounding surfaces at an average temperature of Tsurr = 295 K. Disregarding any heat loss through the upper part of the iron. (a) express the differential equation and the boundary conditions for steady one- dimensional heat conduction through the plate, (b) obtain a relation for the temperature of the outer surface of the plate by solving the differential equation, and (c) evaluate the outer surface temperature.

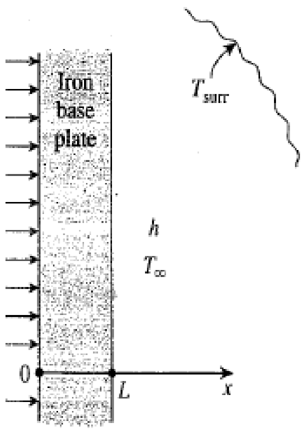


FIGURE P2–138

(a)

Find the heat flux through the inner surface of the base plate

$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}}$$

Here, $\dot{Q}_0$ is amount of heat on left side of the base plate and A_{base} is base area of the base plate of the iron.

Substitute 1200 W for $\dot{Q}_0$ and $150\times10^{-4}\text{ m}^2$ for A_{base}.

$$\begin{aligned}\dot{q}_0 &= \frac{1200}{150\times10^{-4}}\\ &= 80,000\text{ W/m}^2\end{aligned}$$

Step 2 of 7

Write the steady state, one dimensional differential heat conduction equation with no internal heat generation,

$$\frac{d^2T}{dx^2} = 0$$

The boundary conditions to solve the differential equations are:

At $x = 0$, the amount of heat generation is equal to the heat conduction

$$-k\left(\frac{dT}{dx}\right)_{x=0} = \dot{q}_0$$

Here, *k* is thermal conductivity of the plate.

The other boundary condition is at $x = L$, the amount of heat conduction in the iron plate is equal to the sum of heat transfers due to convection and radiation.

$$\begin{aligned}-k\left(\frac{dT}{dx}\right)_{x=L} &= Q_{\text{conv}} + Q_{\text{rad}}\\ -k\left(\frac{dT}{dx}\right)_{x=L} &= h\left[T_{x=L} - T_\infty\right] + \varepsilon\sigma\left[T^4{}_{x=L} - T^4_{\text{surr}}\right]\\ -k\left(\frac{dT}{dx}\right)_{x=L} &= h\left[T_2 - T_\infty\right] + \varepsilon\sigma\left[T^4{}_2 - T^4_{\text{surr}}\right]\end{aligned}$$

Here, *h* is heat transfer coefficient, T_∞ is ambient air temperature, T_{surr} is surrounding air temperature, T_2 is temperature on the outer surface of the plate, ε is emissivity and σ is Stefan Boltzmann's constant.

Therefore, the two boundary conditions to solve the differential heat conduction is

$\boxed{-k\left(\frac{dT}{dx}\right)_{x=0} = \dot{q}_0}$ and

$$\boxed{-k\left(\frac{dT}{dx}\right)_{x=L} = h\left[T_2 - T_\infty\right] + \varepsilon\sigma\left[T^4{}_2 - T^4_{\text{surr}}\right]}$$

(b)

Integrate the differential heat conduction equation twice.

$$\int \frac{d^2T}{dx^2} = 0$$
$$\frac{dT}{dx} = C_1$$
$$\int \frac{dT}{dx} = \int C_1$$
$$T(x) = C_1 x + C_2$$

Here, C_1 and C_2 are arbitrary constants.

Step 4 of 7

Apply the two boundary conditions and solve the arbitrary constants in the equation,

$$T(x) = C_1 x + C_2$$

Consider the following first boundary condition at $x = 0$.

$$-k\left(\frac{dT}{dx}\right)_{x=0} = \dot{q}_0$$
$$-kC_1 = \dot{q}_0$$
$$C_1 = -\frac{\dot{q}_0}{k}$$

Consider the following Second boundary condition at $x = L$

$$-k\left(\frac{dT}{dx}\right)_{x=L} = h\left[T_2 - T_\infty\right] + \varepsilon\sigma\left[T^4{}_2 - T_{\text{surr}}^4\right]$$
$$-kC_1 = h\left[T_2 - T_\infty\right] + \varepsilon\sigma\left[\left(T_2\right)^4 - T_{\text{surr}}^4\right]$$

Step 5 of 7

Eliminate the constant *C*1 from the two relations above gives the following expression for the outer surface temperature *T*2.

$$-kC_1 = h\left[T_2 - T_\infty\right] + \varepsilon\sigma\left[\left(T_2\right)^4 - T_{\text{surr}}^4\right]$$
$$-k \times \left(-\frac{\dot{q}_0}{k}\right) = h\left[T_2 - T_\infty\right] + \varepsilon\sigma\left[\left(T_2\right)^4 - T_{\text{surr}}^4\right]$$
$$\dot{q}_0 = h\left[T_2 - T_\infty\right] + \varepsilon\sigma\left[\left(T_2\right)^4 - T_{\text{surr}}^4\right]$$

Step 6 of 7

Therefore, the temperature of the outer surface of the plate is $\boxed{\begin{array}{l} h\left[T_2 - T_\infty\right] + \\ \varepsilon\sigma\left[\left(T_2\right)^4 - T_{\text{surr}}^4\right] \end{array}}$

(c)

Find the outer surface temperature by using the equation,

$$\dot{q}_0 = h[T_2 - T_\infty] + \varepsilon\sigma\left[(T_2)^4 - T_{\text{surr}}^4\right]$$

Substitute $30\,\text{W/m}^2\cdot\text{K}$ for *h*, $26°\text{C}$ for T_∞, 0.7 for ε, $5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4$ for σ, $295\,\text{K}$ for T_{surr}

$$\dot{q}_0 = h[T_2 - T_\infty] + \varepsilon\sigma\left[(T_2 + 273)^4 - T_{\text{surr}}^4\right]$$

$$80000 = 30\ (T_2 - 26) + 0.7\left(5.67\times10^{-8}\right)\left[(T_2 + 273)^4 - 295^4\right]$$

Solve the equation for temperature,

$$T_2 = 819.306°\text{C}$$

Therefore, outer surface temperature of the plate is $\boxed{819.306°\text{C}}$