Fluid Mechanics: Fundamentals and Applications, 2nd Edition
Yunus A. Cengel, John M. Cimbala
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Chapter 2
PROPERTIES OF FLUIDS

Lecture slides by

Hasan Hacışevki

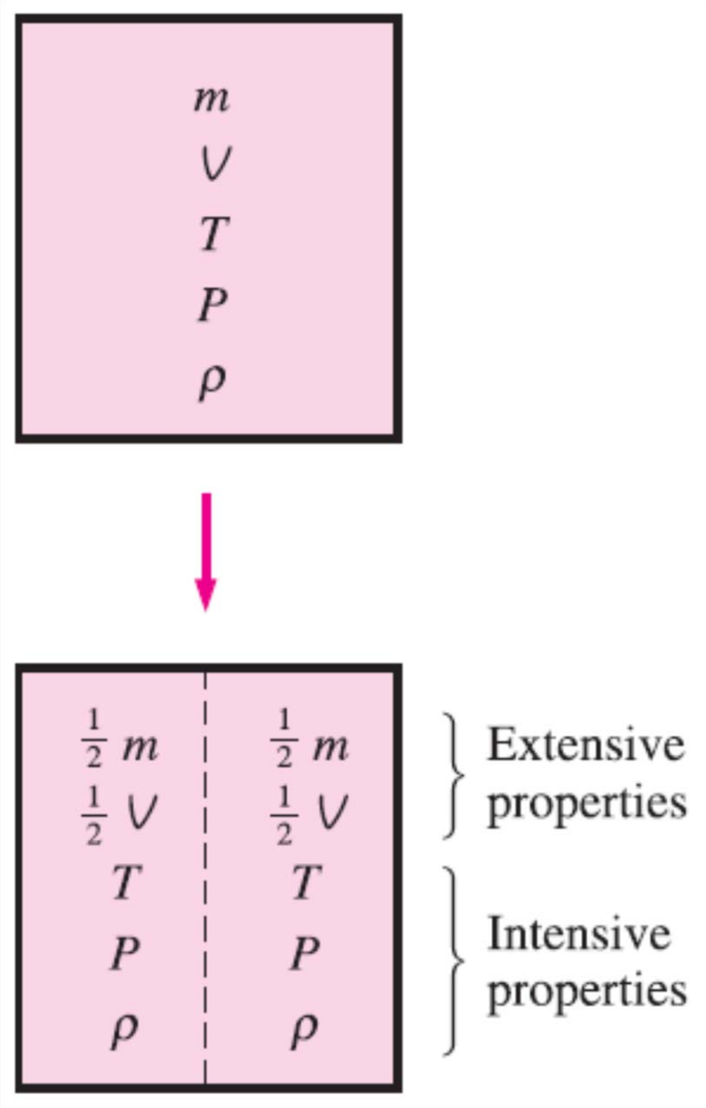
A drop forms when liquid is forced out of a small tube. The shape of the drop is determined by a balance of pressure, gravity, and surface tension forces.

Objectives

- Have a working knowledge of the basic properties of fluids and understand the continuum approximation.
- Have a working knowledge of viscosity and the consequences of the frictional effects it causes in fluid flow.
- Calculate the capillary rise (or drop) in tubes due to the surface tension effect.

2–1 ■ INTRODUCTION

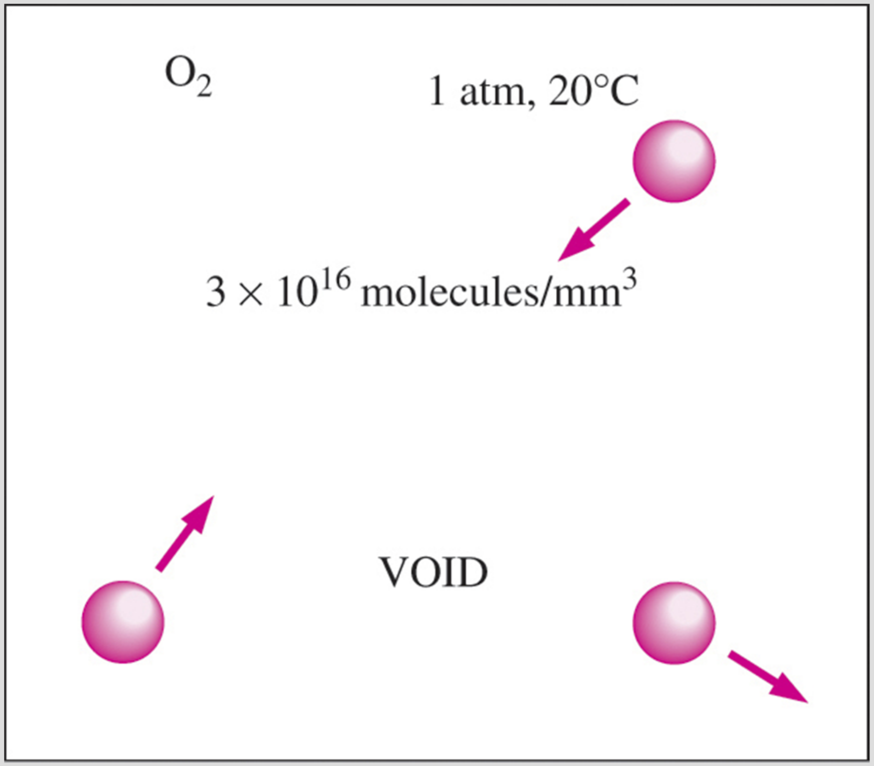
- **Property:** Any characteristic of a system.
- Some familiar properties are pressure *P*, temperature *T*, volume *V*, and mass *m*.
- Properties are considered to be either *intensive* or *extensive*.
- **Intensive properties:** Those that are independent of the mass of a system, such as temperature, pressure, and density.
- **Extensive properties:** Those whose values depend on the size—or extent—of the system.
- **Specific properties:** Extensive properties per unit mass.



Criterion to differentiate intensive and extensive properties.

Continuum

- Matter is made up of atoms that are widely spaced in the gas phase. Yet it is very convenient to disregard the atomic nature of a substance and view it as a continuous, homogeneous matter with no holes, that is, a **continuum**.
- The continuum idealization allows us to treat properties as point functions and to assume the properties vary continually in space with no jump discontinuities.
- This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules.
- This is the case in practically all problems.
- In this text we will limit our consideration to substances that can be modeled as a continuum.



Despite the relatively large gaps between molecules, a substance can be treated as a continuum because of the very large number of molecules even in an extremely small volume.

The length scale associated with most flows, such as seagulls in flight, is orders of magnitude larger than the mean free path of the air molecules. Therefore, here, and for all fluid flows considered in this book, the continuum idealization is appropriate.

2–2 ■ DENSITY AND SPECIFIC GRAVITY

Density

$$\rho = \frac{m}{V} \qquad (\mathrm{kg/m^3})$$

Specific volume

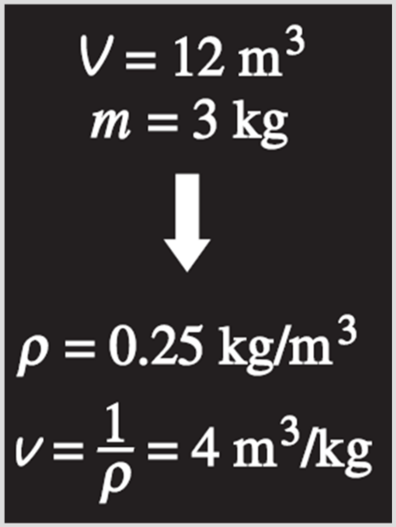
$$v = \frac{V}{m} = \frac{1}{\rho}$$

Specific gravity: The ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C).

$$\mathrm{SG} = \frac{\rho}{\rho_{\mathrm{H_2O}}}$$

Specific weight: The weight of a unit volume of a substance.

$$\gamma_s = \rho g \qquad (\mathrm{N/m^3})$$



$$V = 12\ \mathrm{m^3}$$
$$m = 3\ \mathrm{kg}$$
$$\rho = 0.25\ \mathrm{kg/m^3}$$
$$v = \frac{1}{\rho} = 4\ \mathrm{m^3/kg}$$

Density is mass per unit volume; specific volume is volume per unit mass.

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

Density of Ideal Gases

Equation of state: Any equation that relates the pressure, temperature, and density (or specific volume) of a substance.

Ideal-gas equation of state: The simplest and best-known equation of state for substances in the gas phase.

$$Pv = RT \qquad \text{or} \qquad P = \rho RT$$

$$R = R_u / M \qquad R_u = 8.314\ \text{kJ/kmol} \cdot \text{K}$$

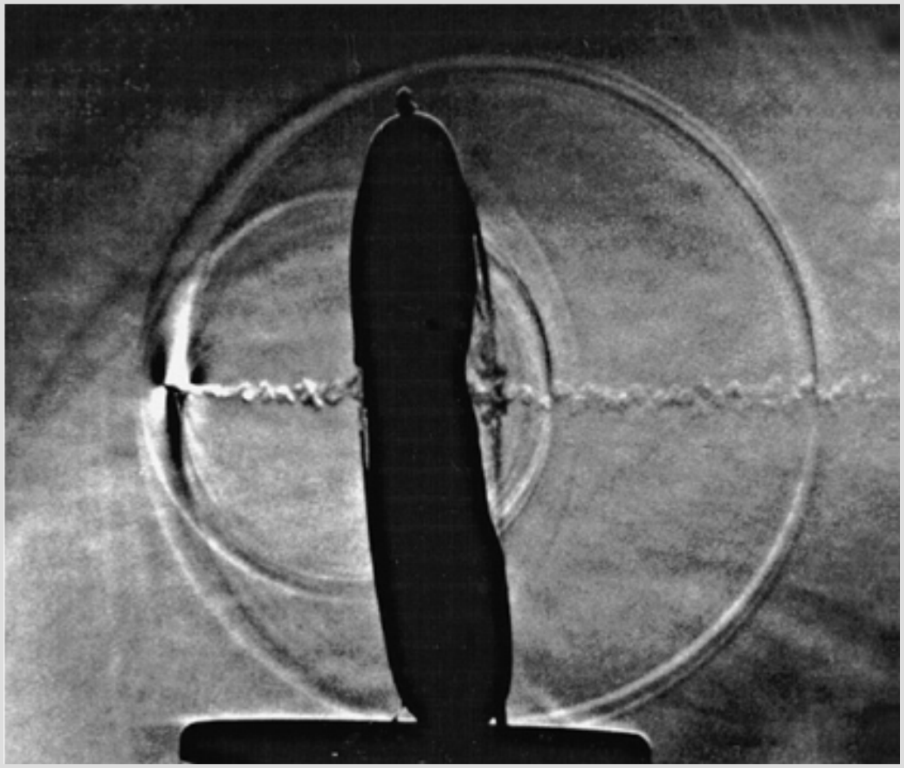
R_u: The universal gas constant

$$PV = mRT \text{ or } PV = NR_uT.$$

The thermodynamic temperature scale in the SI is the **Kelvin scale.**
In the English system, it is the **Rankine scale.**

$$T(\text{K}) = T(^\circ\text{C}) + 273.15 = T(R)/1.8$$

$$T(\text{R}) = T(^\circ\text{F}) + 459.67 = 1.8\, T(\text{K})$$



Air behaves as an ideal gas, even at very high speeds. In this schlieren image, a bullet traveling at about the speed of sound bursts through both sides of a balloon, forming two expanding shock waves. The turbulent wake of the bullet is also visible.

An ideal gas is a hypothetical substance that obeys the relation $Pv = RT$.

The ideal-gas relation closely approximates the *P-v-T* behavior of real gases at low densities.

At low pressures and high temperatures, the density of a gas decreases and the gas behaves like an ideal gas.

In the range of practical interest, many familiar gases such as air, nitrogen, oxygen, hydrogen, helium, argon, neon, and krypton and even heavier gases such as carbon dioxide can be treated as ideal gases with negligible error.

Dense gases such as water vapor in steam power plants and refrigerant vapor in refrigerators, however, should not be treated as ideal gases since they usually exist at a state near saturation.

EXAMPLE 2–1 Density, Specific Gravity, and Mass of Air in a Room

Determine the density, specific gravity, and mass of the air in a room whose dimensions are 4 m × 5 m × 6 m at 100 kPa and 25°C (Fig. 2–4).

Solution The density, specific gravity, and mass of the air in a room are to be determined.

Assumptions At specified conditions, air can be treated as an ideal gas.

Properties The gas constant of air is $R = 0.287\ \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$.

Analysis The density of the air is determined from the ideal-gas relation $P = \rho RT$ to be

$$\rho = \frac{P}{RT} = \frac{100\ \text{kPa}}{(0.287\ \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273.15)\ \text{K}} = \mathbf{1.17\ kg/m^3}$$

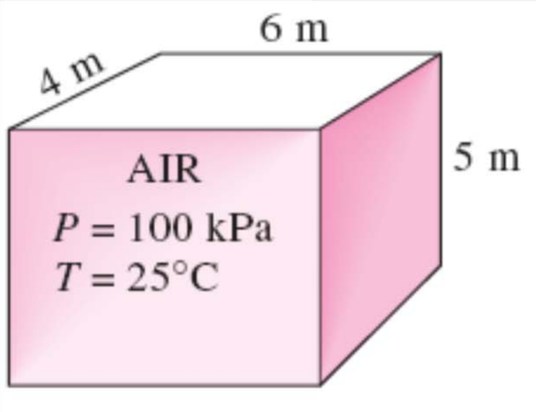
Then the specific gravity of the air becomes

$$\text{SG} = \frac{\rho}{\rho_{\text{H}_2\text{O}}} = \frac{1.17\ \text{kg/m}^3}{1000\ \text{kg/m}^3} = \mathbf{0.00117}$$

Finally, the volume and the mass of the air in the room are

$$V = (4\ \text{m})(5\ \text{m})(6\ \text{m}) = 120\ \text{m}^3$$

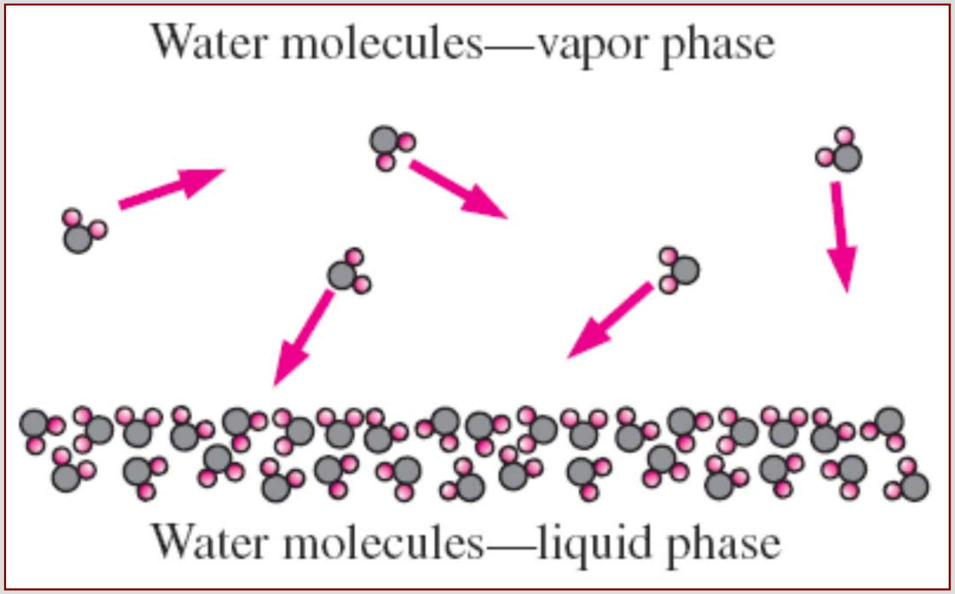
$$m = \rho V = (1.17\ \text{kg/m}^3)(120\ \text{m}^3) = \mathbf{140\ kg}$$



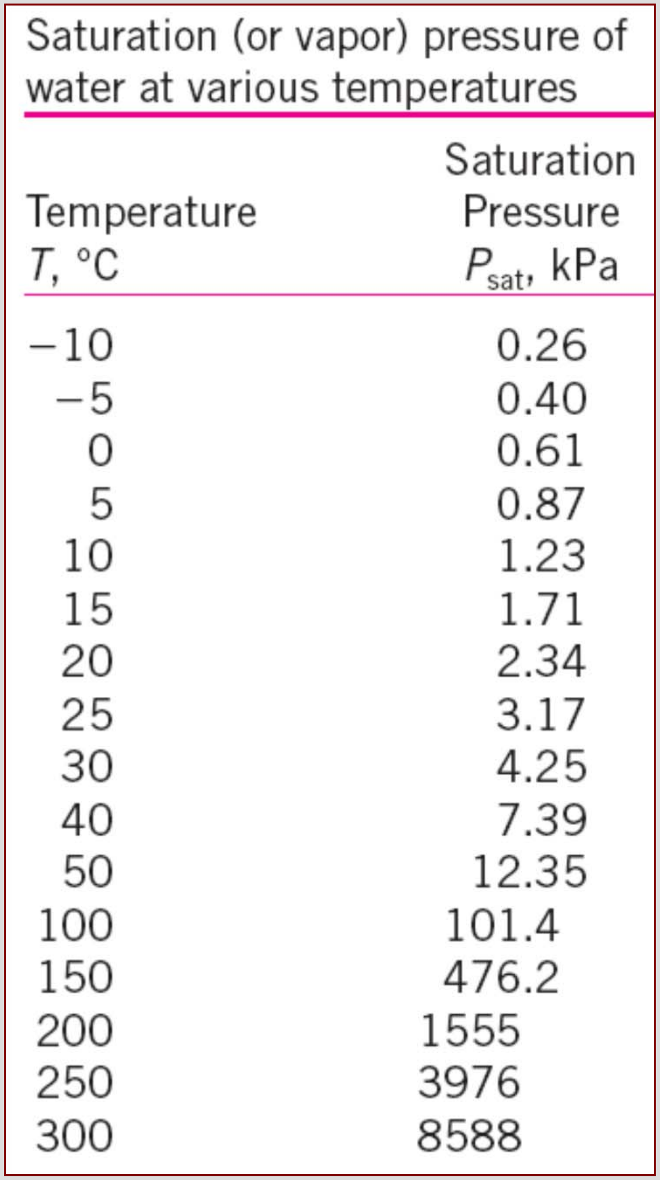
Discussion Note that we converted the temperature to the unit K from °C before using it in the ideal-gas relation.

2–3 ■ VAPOR PRESSURE AND CAVITATION

- **Saturation temperature T_{sat}**: The temperature at which a pure substance changes phase at a given pressure.
- **Saturation pressure P_{sat}**: The pressure at which a pure substance changes phase at a given temperature.
- **Vapor pressure (P_v):** The pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature. It is identical to the saturation pressure P_{sat} of the liquid ($P_v = P_{sat}$).
- **Partial pressure:** The pressure of a gas or vapor in a mixture with other gases. For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor.

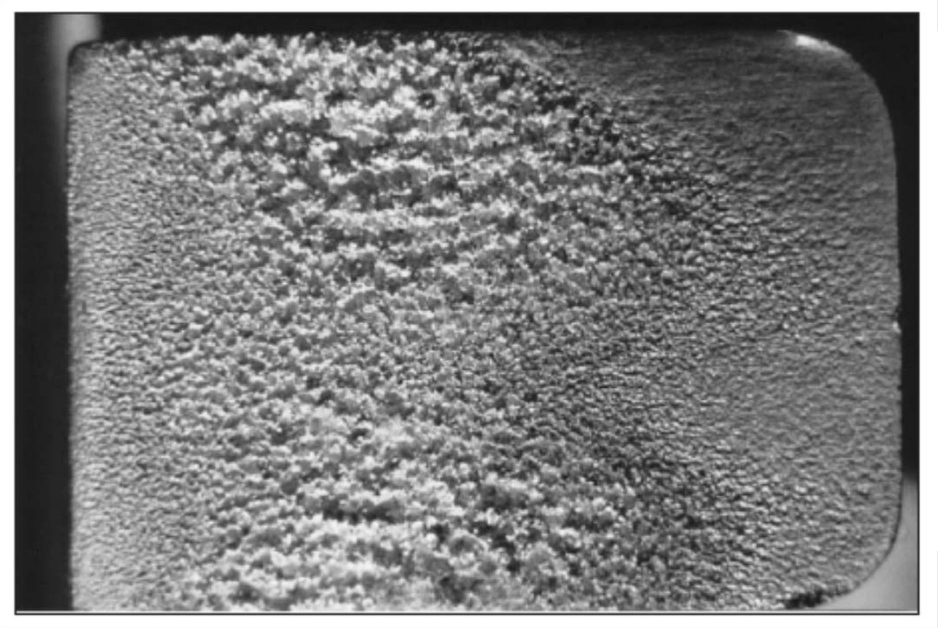


The vapor pressure (saturation pressure) of a pure substance (e.g., water) is the pressure exerted by its vapor molecules when the system is in phase equilibrium with its liquid molecules at a given temperature.

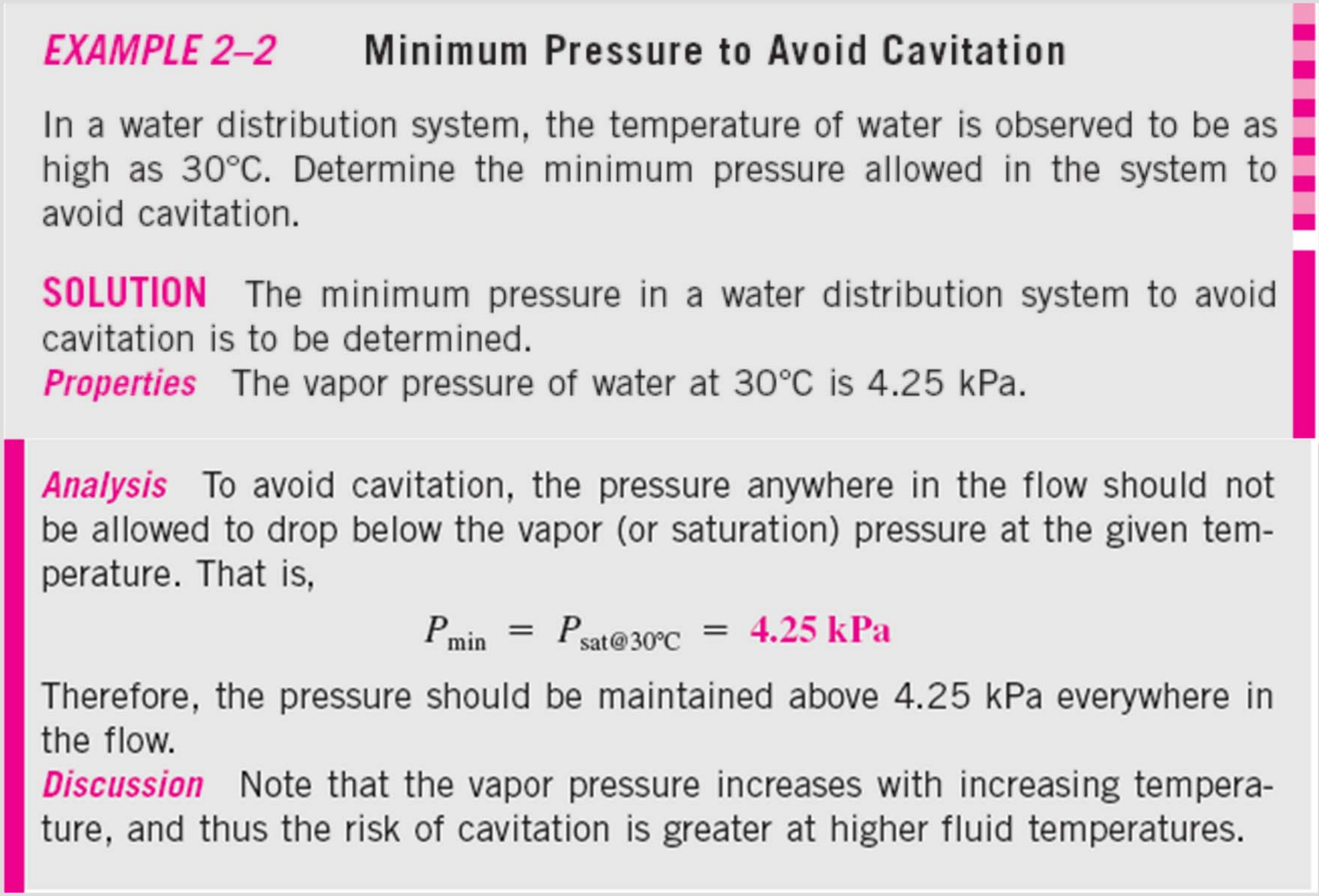

Saturation (or vapor) pressure of water at various temperatures

Temperature T, °C	Saturation Pressure P_{sat}, kPa
−10	0.26
−5	0.40
0	0.61
5	0.87
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.39
50	12.35
100	101.4
150	476.2
200	1555
250	3976
300	8588

- There is a possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization.
- The vapor bubbles (called **cavitation bubbles** since they form “cavities” in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.
- This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called **cavitation**, and it is an important consideration in the design of hydraulic turbines and pumps.



Cavitation damage on a 16-mm by 23-mm aluminum sample tested at 60 m/s for 2.5 h. The sample was located at the cavity collapse region downstream of a cavity generator specifically designed to produce high damage potential.



EXAMPLE 2–2 **Minimum Pressure to Avoid Cavitation**

In a water distribution system, the temperature of water is observed to be as high as 30°C. Determine the minimum pressure allowed in the system to avoid cavitation.

SOLUTION The minimum pressure in a water distribution system to avoid cavitation is to be determined.

Properties The vapor pressure of water at 30°C is 4.25 kPa.

Analysis To avoid cavitation, the pressure anywhere in the flow should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\text{min}} = P_{\text{sat@30°C}} = \mathbf{4.25\ kPa}$$

Therefore, the pressure should be maintained above 4.25 kPa everywhere in the flow.

Discussion Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

2–4 ■ ENERGY AND SPECIFIC HEATS

- Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear, and their sum constitutes the **total energy, *E*** of a system.
- Thermodynamics deals only with the ***change*** of the total energy.
- **Macroscopic forms of energy**: Those a system possesses as a whole with respect to some outside reference frame, such as kinetic and potential energies.
- **Microscopic forms of energy**: Those related to the molecular structure of a system and the degree of the molecular activity.
- **Internal energy, *U***: The sum of all the microscopic forms of energy.
- **Kinetic energy, KE**: The energy that a system possesses as a result of its motion relative to some reference frame.
- **Potential energy, PE:** The energy that a system possesses as a result of its elevation in a gravitational field.

The macroscopic energy of an object changes with velocity and elevation.

$$h = u + Pv = u + \frac{P}{\rho}$$ Enthalpy

Energy of a flowing fluid

$$e_{\text{flowing}} = P/\rho + e = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz \qquad (\text{kJ/kg})$$

$$du = c_v\, dT \qquad \text{and} \qquad dh = c_p\, dT$$

$$\Delta u \cong c_{v,\text{avg}}\, \Delta T \qquad \text{and} \qquad \Delta h \cong c_{p,\text{avg}}\, \Delta T$$

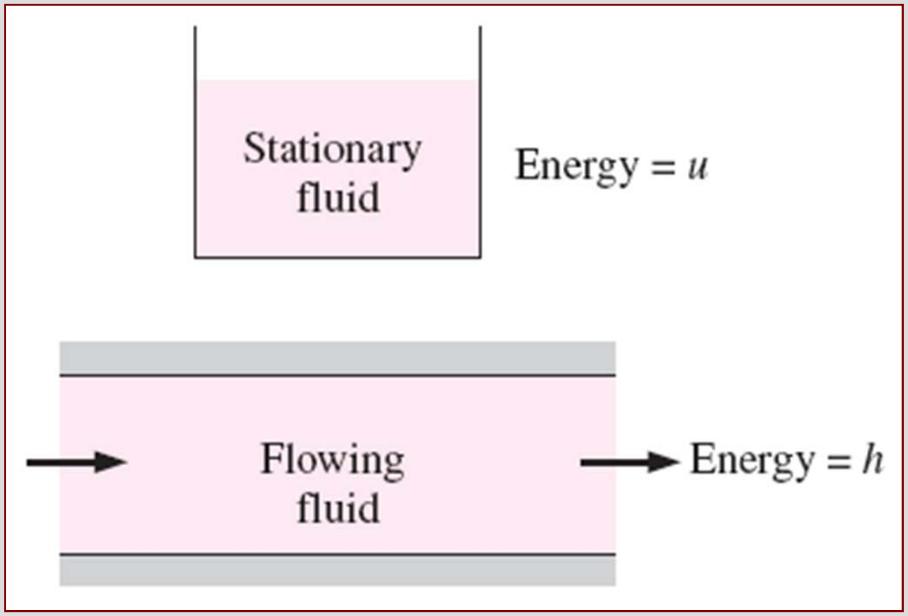
$$\Delta h = \Delta u + \Delta P/\rho \cong c_{\text{avg}}\, \Delta T + \Delta P/\rho$$

$$\Delta h = \Delta u \cong c_{\text{avg}}\, \Delta T$$ for a P = const. process

$$\Delta h = \Delta P/\rho$$ For a T = const. process

P/ρ is the *flow energy*, also called the *flow work*, which is the energy per unit mass needed to move the fluid and maintain flow.

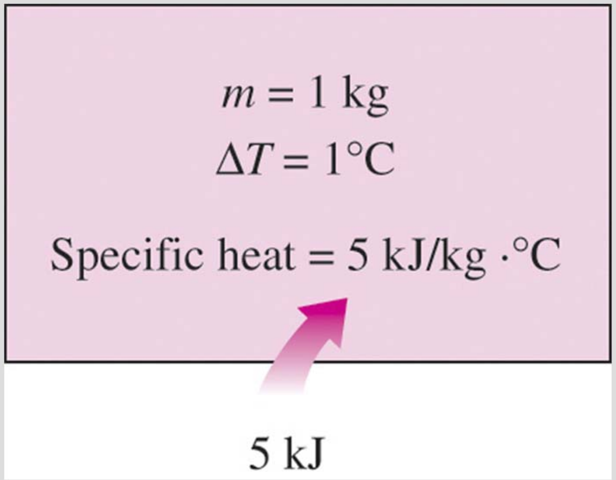
The *internal energy u* represents the microscopic energy of a nonflowing fluid per unit mass, whereas *enthalpy h* represents the microscopic energy of a flowing fluid per unit mass.



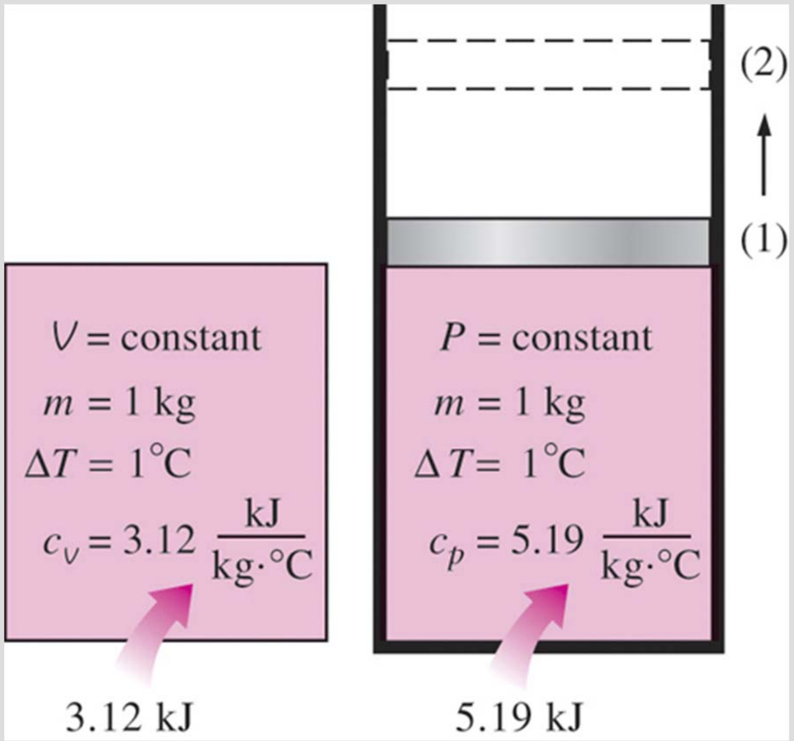
Specific Heats

Specific heat at constant volume, c_v: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.



Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.



Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).

2–5 ■ COMPRESSIBILITY AND SPEED OF SOUND

Coefficient of Compressibility

We know from experience that the volume (or density) of a fluid changes with a change in its temperature or pressure.

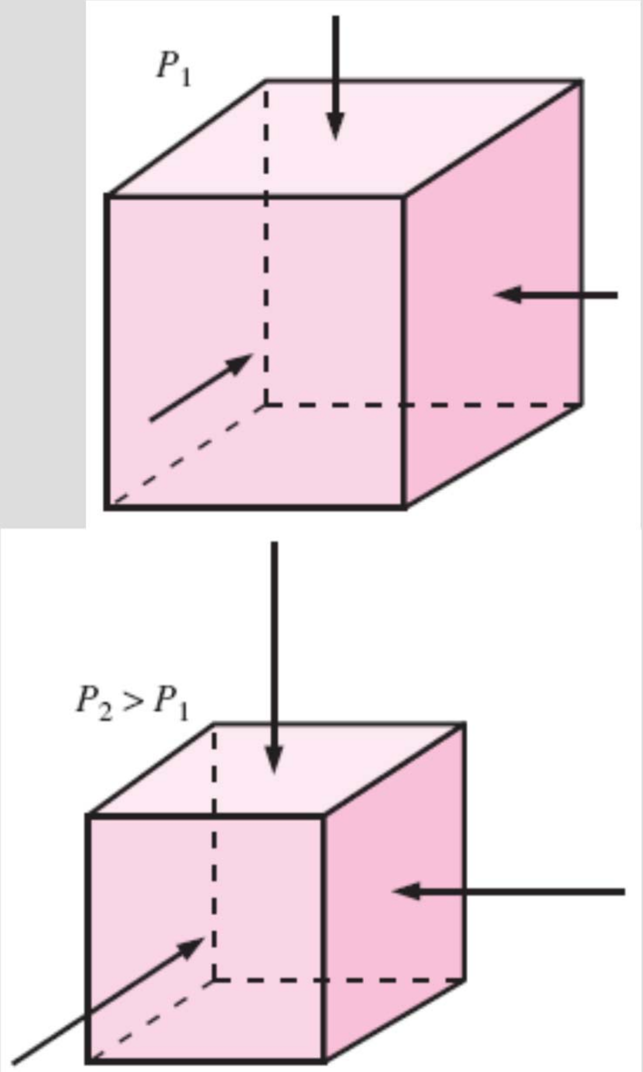
Fluids usually expand as they are heated or depressurized and contract as they are cooled or pressurized.

But the amount of volume change is different for different fluids, and we need to define properties that relate volume changes to the changes in pressure and temperature.

Two such properties are:

the bulk modulus of elasticity κ

the coefficient of volume expansion β.



Fluids, like solids, compress when the applied pressure is increased from P_1 to P_2.

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \qquad \text{(Pa)}$$

$$\kappa \cong -\frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta \rho/\rho} \qquad (T = \text{constant})$$

Coefficient of compressibility (also called the **bulk modulus of compressibility** or **bulk modulus of elasticity**) for fluids

The coefficient of compressibility represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant.

What is the coefficient of compressibility of a truly incompressible substance (v = constant)?

A large value of κ indicates that a large change in pressure is needed to cause a small fractional change in volume, and thus a fluid with a large κ is essentially incompressible.

This is typical for liquids, and explains why liquids are usually considered to be *incompressible*.

Water hammer: Characterized by a sound that resembles the sound produced when a pipe is “hammered.” This occurs when a liquid in a piping network encounters an abrupt flow restriction (such as a closing valve) and is locally compressed.

The acoustic waves that are produced strike the pipe surfaces, bends, and valves as they propagate and reflect along the pipe, causing the pipe to vibrate and produce the familiar sound.

Water hammering can be quite destructive, leading to leaks or even structural damage. The effect can be suppressed with a *water hammer arrestor*.

Water hammer arrestors: (a) A large surge tower built to protect the pipeline against water hammer damage. (b) Much smaller arrestors used for supplying water to a household washing machine.

For an ideal gas, $P = \rho RT$ and $(\partial P/\partial \rho)_T = RT = P/\rho$, and thus

$$\kappa_{\text{ideal gas}} = P \qquad \text{(Pa)}$$

The coefficient of compressibility of an ideal gas is equal to its absolute pressure, and the coefficient of compressibility of the gas increases with increasing pressure.

Ideal gas:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta P}{P} \qquad (T = \text{constant})$$

The percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure.

Isothermal compressibility: The inverse of the coefficient of compressibility.

The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure.

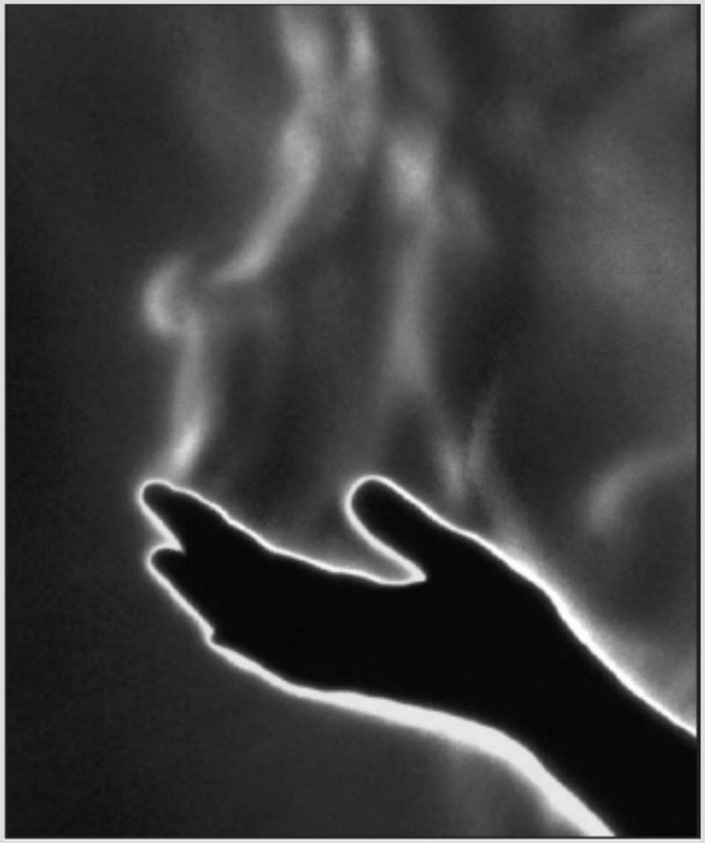
$$\alpha = \frac{1}{\kappa} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T \qquad \text{(1/Pa)}$$

Coefficient of Volume Expansion

The density of a fluid depends more strongly on temperature than it does on pressure.

The variation of density with temperature is responsible for numerous natural phenomena such as winds, currents in oceans, rise of plumes in chimneys, the operation of hot-air balloons, heat transfer by natural convection, and even the rise of hot air and thus the phrase "heat rises".

To quantify these effects, we need a property that represents the *variation of the density of a fluid with temperature at constant pressure*.



Natural convection over a woman's hand.

The **coefficient of volume expansion** (or *volume expansivity*): The variation of the density of a fluid with temperature at constant pressure.

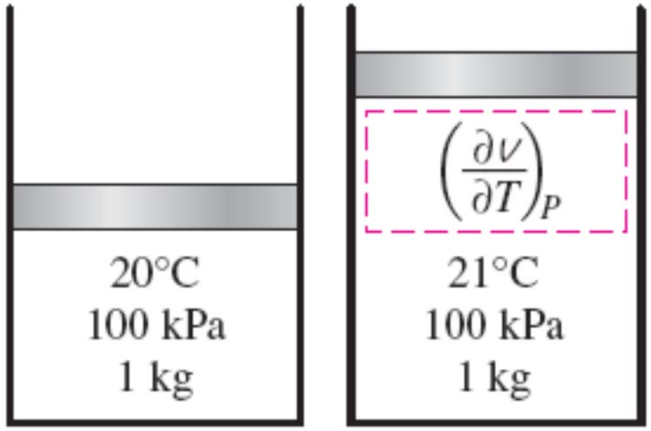
$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\mathrm{K})$$

$$\beta \approx \frac{\Delta v/v}{\Delta T} = -\frac{\Delta \rho/\rho}{\Delta T} \qquad (\text{at constant } P)$$

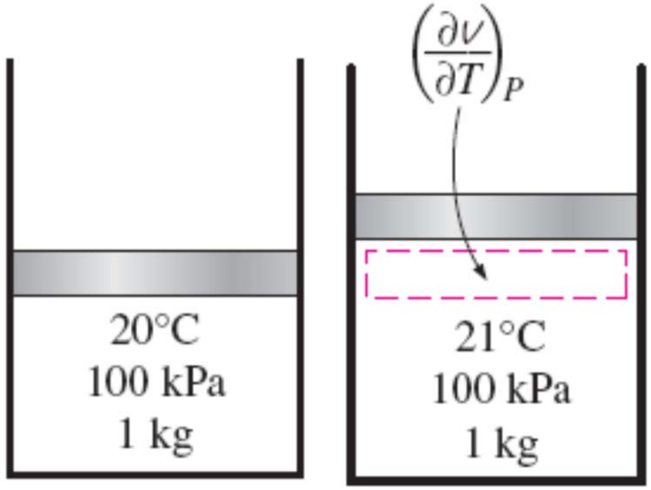
A large value of β for a fluid means a large change in density with temperature, and the product $\beta \Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change of *T* at constant pressure.

The volume expansion coefficient of an *ideal gas* ($P = \rho RT$) at a temperature *T* is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\mathrm{K})$$



(*a*) A substance with a large β



(*b*) A substance with a small β

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

In the study of natural convection currents, the condition of the main fluid body that surrounds the finite hot or cold regions is indicated by the subscript “infinity” to serve as a reminder that this is the value at a distance where the presence of the hot or cold region is not felt. In such cases, the volume expansion coefficient can be expressed approximately as

$$\beta \approx -\frac{(\rho_\infty - \rho)/\rho}{T_\infty - T} \qquad \text{or} \qquad \rho_\infty - \rho = \rho\beta(T - T_\infty)$$

The combined effects of pressure and temperature changes on the volume change of a fluid can be determined by taking the specific volume to be a function of *T* and *P*.

$$dv = \left(\frac{\partial v}{\partial T}\right)_P dT + \left(\frac{\partial v}{\partial P}\right)_T dP = (\beta\, dT - \alpha\, dP)v$$

The fractional change in volume (or density) due to changes in pressure and temperature can be expressed approximately as

$$\frac{\Delta v}{v} = -\frac{\Delta\rho}{\rho} \cong \beta\,\Delta T - \alpha\,\Delta P$$

EXAMPLE 2–3 **Variation of Density with Temperature and Pressure**

Consider water initially at 20°C and 1 atm. Determine the final density of the water (*a*) if it is heated to 50°C at a constant pressure of 1 atm, and (*b*) if it is compressed to 100-atm pressure at a constant temperature of 20°C. Take the isothermal compressibility of water to be $\alpha = 4.80 \times 10^{-5}$ atm^{-1}.

SOLUTION Water at a given temperature and pressure is considered. The densities of water after it is heated and after it is compressed are to be determined.

Assumptions **1** The coefficient of volume expansion and the isothermal compressibility of water are constant in the given temperature range. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes.

Properties The density of water at 20°C and 1 atm pressure is $\rho_1 =$ 998.0 kg/m^3. The coefficient of volume expansion at the average temperature of (20 + 50)/2 = 35°C is $\beta = 0.337 \times 10^{-3}$ K^{-1}. The isothermal compressibility of water is given to be $\alpha = 4.80 \times 10^{-5}$ atm^{-1}.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as (Eq. 2–23)

$$\Delta\rho = \alpha\rho\,\Delta P - \beta\rho\,\Delta T$$

(*a*) The change in density due to the change of temperature from 20°C to 50°C at constant pressure is

$$\Delta\rho = -\beta\rho\,\Delta T = -(0.337 \times 10^{-3}\,\text{K}^{-1})(998\ \text{kg/m}^3)(50 - 20)\ \text{K}$$
$$= -10.0\ \text{kg/m}^3$$

Noting that $\Delta\rho = \rho_2 - \rho_1$, the density of water at 50°C and 1 atm is

$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + (-10.0) = \mathbf{988.0\ kg/m^3}$$

which is almost identical to the listed value of 988.1 kg/m^3 at 50°C in Table A–3. This is mostly due to β varying with temperature almost linearly, as shown in Fig. 2–14.

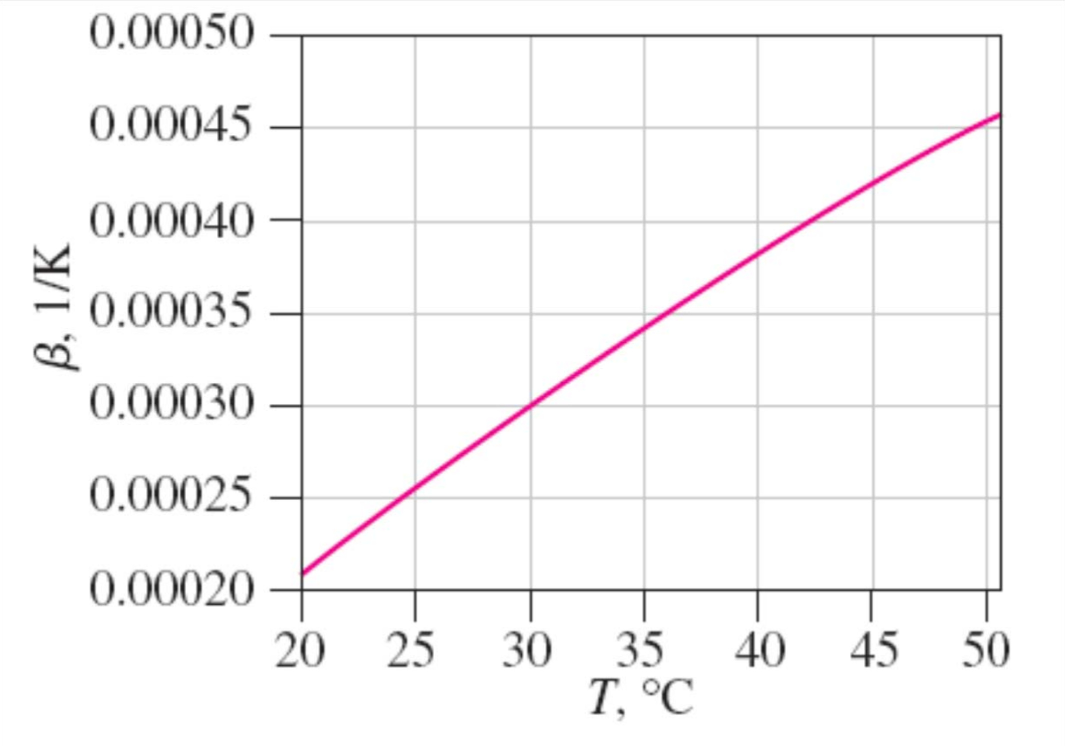
(*b*) The change in density due to a change of pressure from 1 atm to 100 atm at constant temperature is

$$\Delta\rho = \alpha\rho\,\Delta P = (4.80 \times 10^{-5}\ \text{atm}^{-1})(998\ \text{kg/m}^3)(100 - 1)\ \text{atm} = 4.7\ \text{kg/m}^3$$

Then the density of water at 100 atm and 20°C becomes

$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + 4.7 = \mathbf{1002.7\ kg/m^3}$$

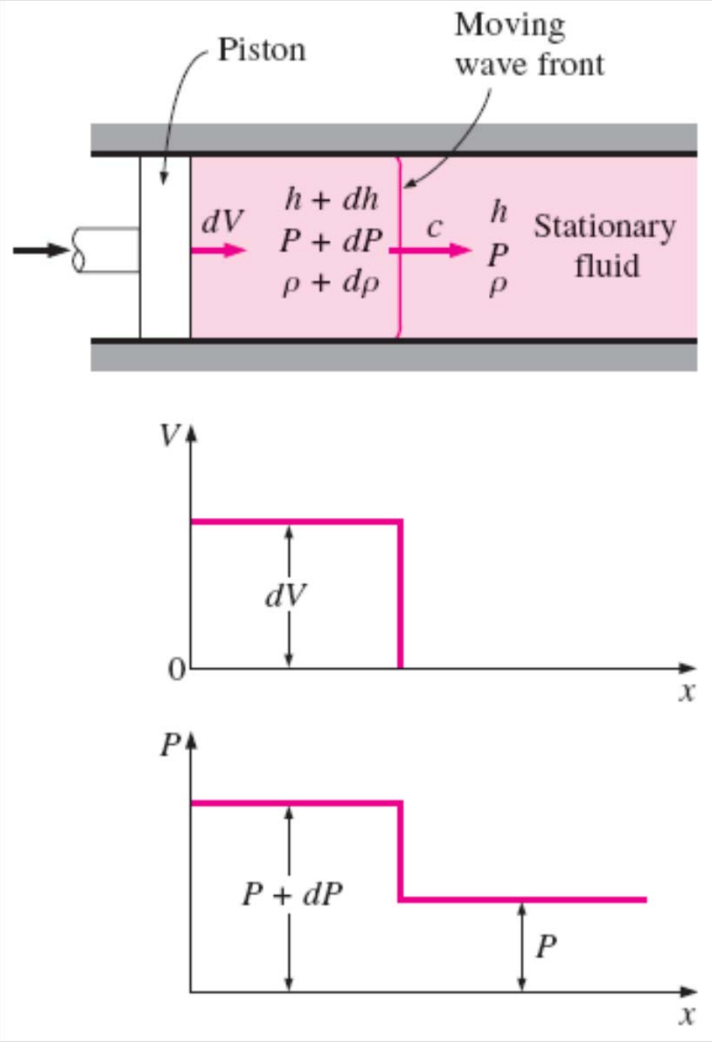
Discussion Note that the density of water decreases while being heated and increases while being compressed, as expected. This problem can be solved more accurately using differential analysis when functional forms of properties are available.



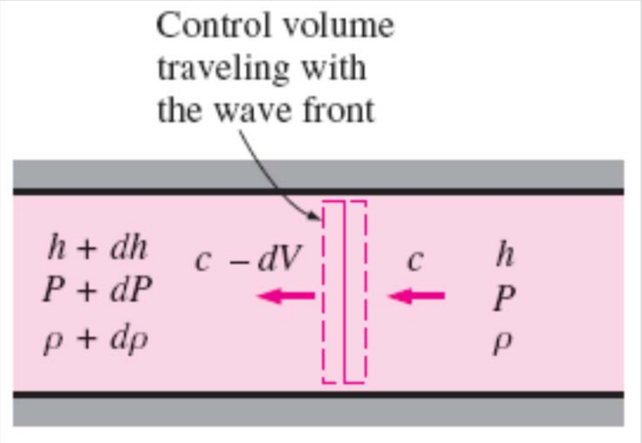
The variation of the coefficient of volume expansion of water with temperature in the range of 20°C to 50°C.

Speed of Sound and Mach Number

Speed of sound (sonic speed): The speed at which an infinitesimally small pressure wave travels through a medium.



Propagation of a small pressure wave along a duct.



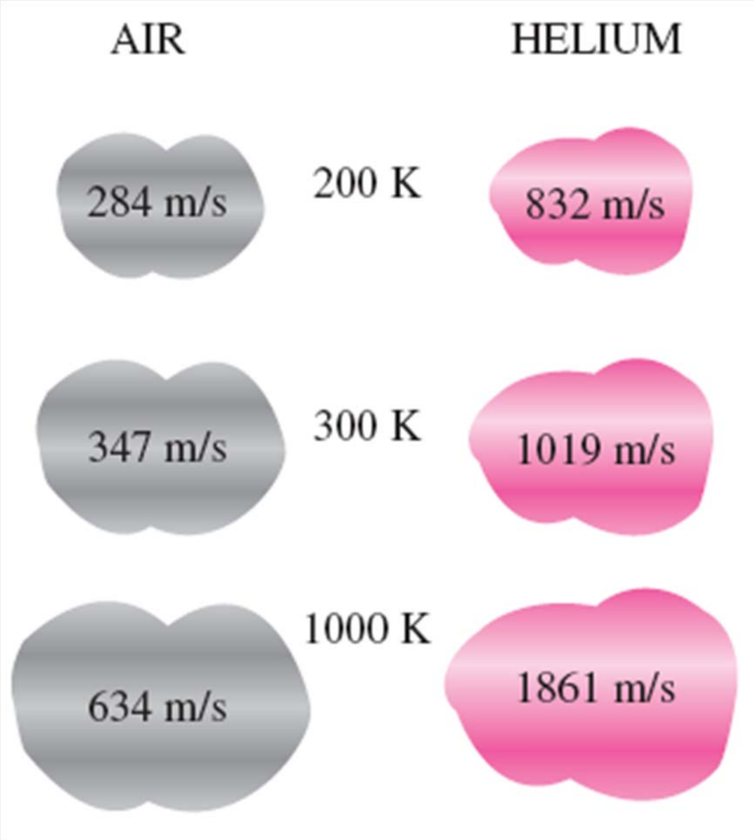
Control volume moving with the small pressure wave along a duct.

$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$$

For any fluid

$$c = \sqrt{kRT}$$

For an ideal gas



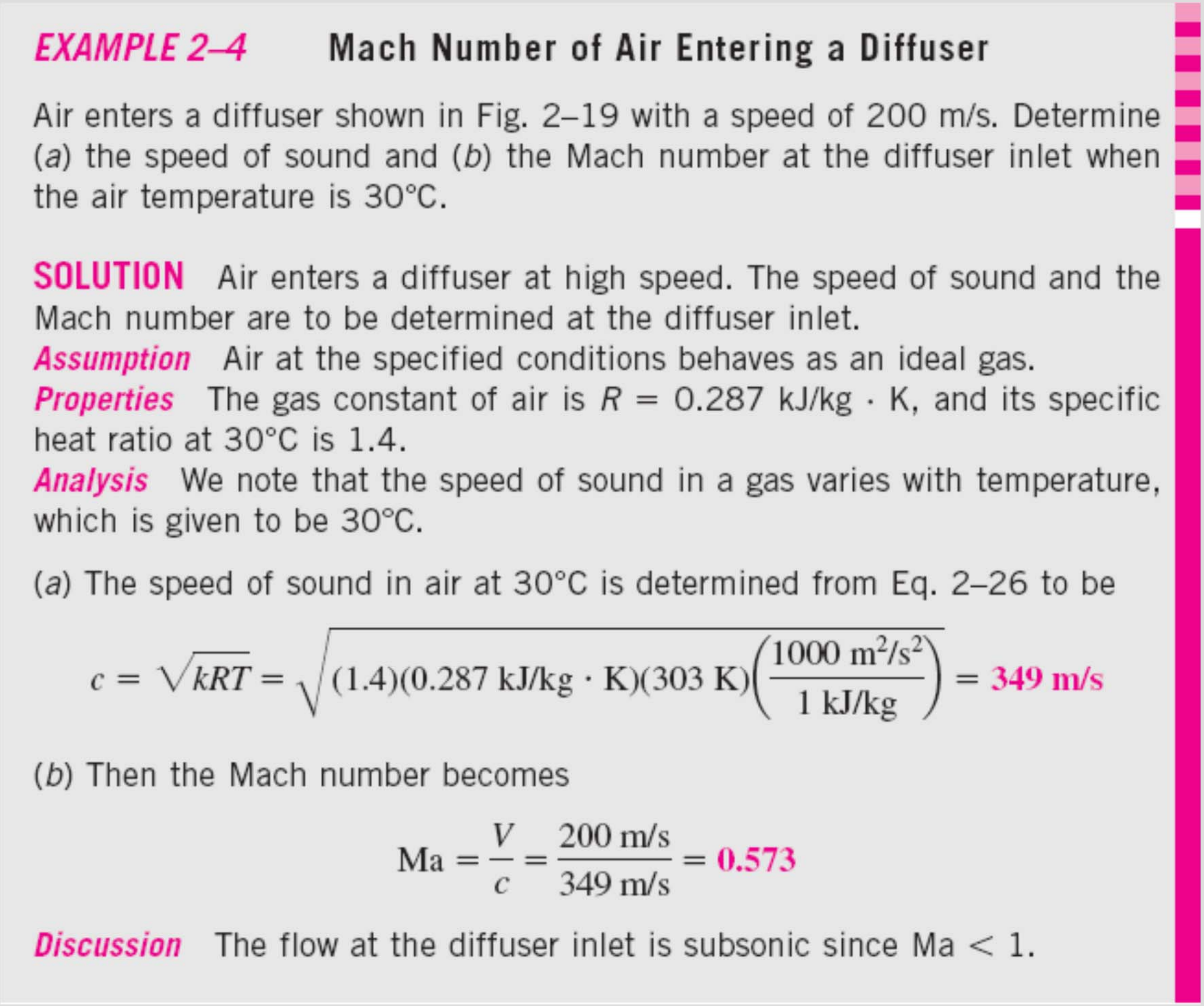
The speed of sound changes with temperature and varies with the fluid.

Mach number Ma: The ratio of the actual speed of the fluid (or an object in still fluid) to the speed of sound in the same fluid at the same state.

$$\mathrm{Ma} = \frac{V}{c}$$

The Mach number depends on the speed of sound, which depends on the state of the fluid.

The Mach number can be different at different temperatures even if the flight speed is the same.



EXAMPLE 2–4 **Mach Number of Air Entering a Diffuser**

Air enters a diffuser shown in Fig. 2–19 with a speed of 200 m/s. Determine (*a*) the speed of sound and (*b*) the Mach number at the diffuser inlet when the air temperature is 30°C.

SOLUTION Air enters a diffuser at high speed. The speed of sound and the Mach number are to be determined at the diffuser inlet.

Assumption Air at the specified conditions behaves as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kJ/kg · K, and its specific heat ratio at 30°C is 1.4.

Analysis We note that the speed of sound in a gas varies with temperature, which is given to be 30°C.

(*a*) The speed of sound in air at 30°C is determined from Eq. 2–26 to be

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(303\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = \mathbf{349\ m/s}$$

(*b*) Then the Mach number becomes

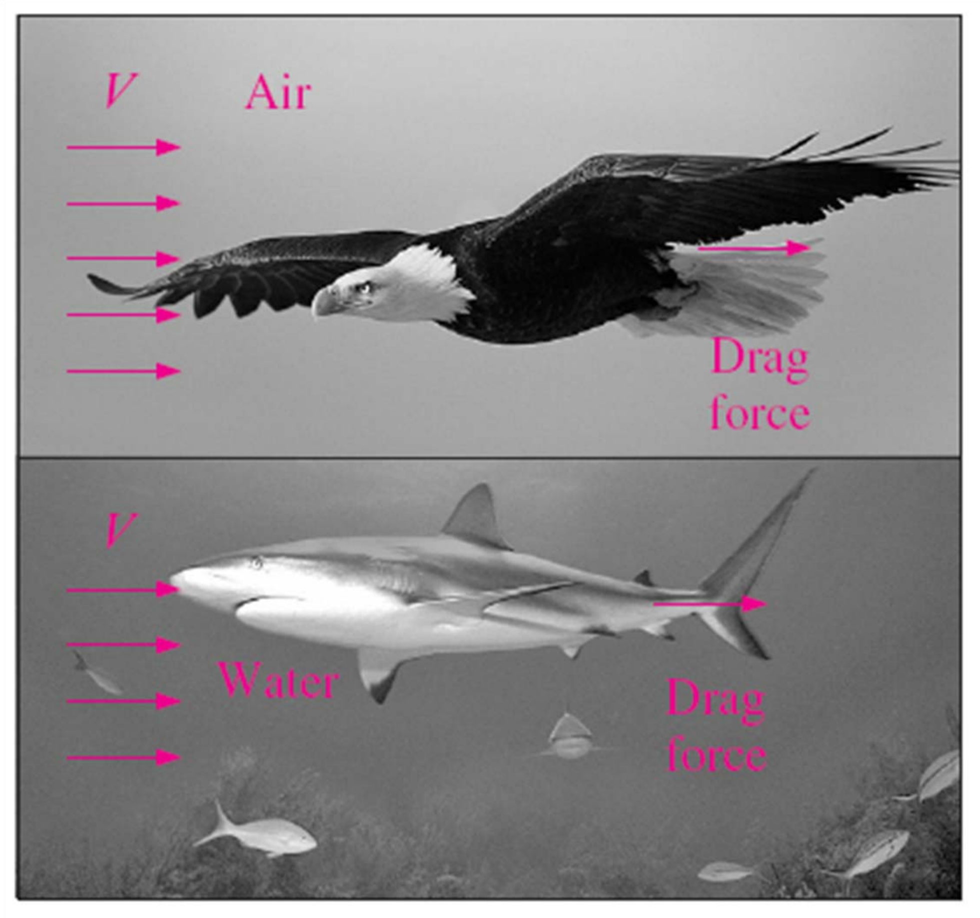
$$\text{Ma} = \frac{V}{c} = \frac{200\ \text{m/s}}{349\ \text{m/s}} = \mathbf{0.573}$$

Discussion The flow at the diffuser inlet is subsonic since Ma < 1.

2–6 ■ VISCOSITY

Viscosity: A property that represents the internal resistance of a fluid to motion or the “fluidity”.

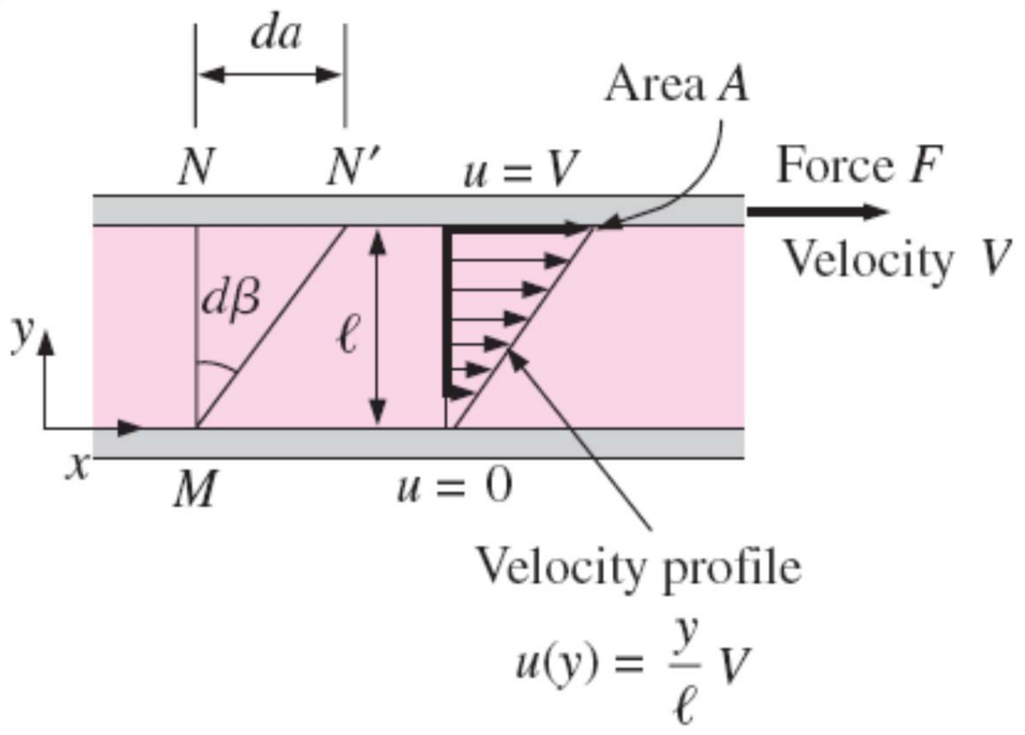
Drag force: The force a flowing fluid exerts on a body in the flow direction. The magnitude of this force depends, in part, on viscosity



The viscosity of a fluid is a measure of its “*resistance to deformation*.”

Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.

A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.



The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

$$\tau = \frac{F}{A}$$

$$u(y) = \frac{y}{\ell} V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell}$$

$$d\beta \approx \tan d\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy} dt$$

$$\frac{d\beta}{dt} = \frac{du}{dy}$$

Newtonian fluids: Fluids for which the rate of deformation is proportional to the shear stress.

$$\tau \propto \frac{d(d\beta)}{dt} \quad \text{or} \quad \tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \qquad (\text{N/m}^2)$$

Shear stress

Shear force

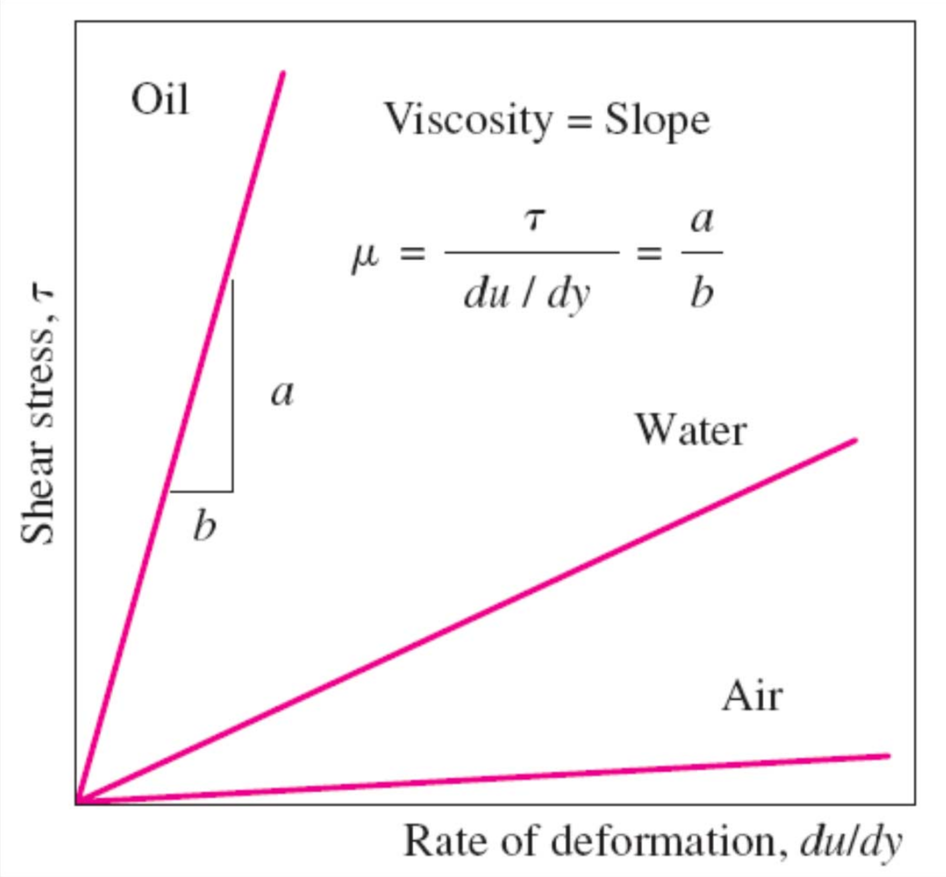
$$F = \tau A = \mu A \frac{du}{dy} \qquad (\text{N})$$

μ coefficient of viscosity

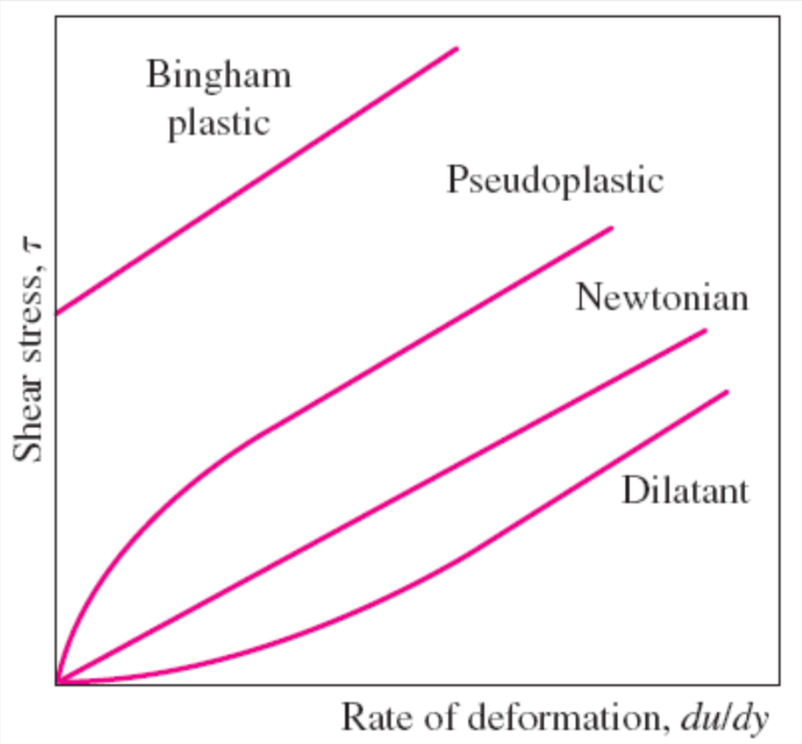
Dynamic (absolute) viscosity

kg/m · s or N · s/m^2 or Pa · s

1 poise = 0.1 Pa · s



The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.



Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).

Kinematic viscosity

$\nu = \mu/\rho$

m^2/s or stoke

1 stoke = 1 cm^2/s

For *liquids*, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures.

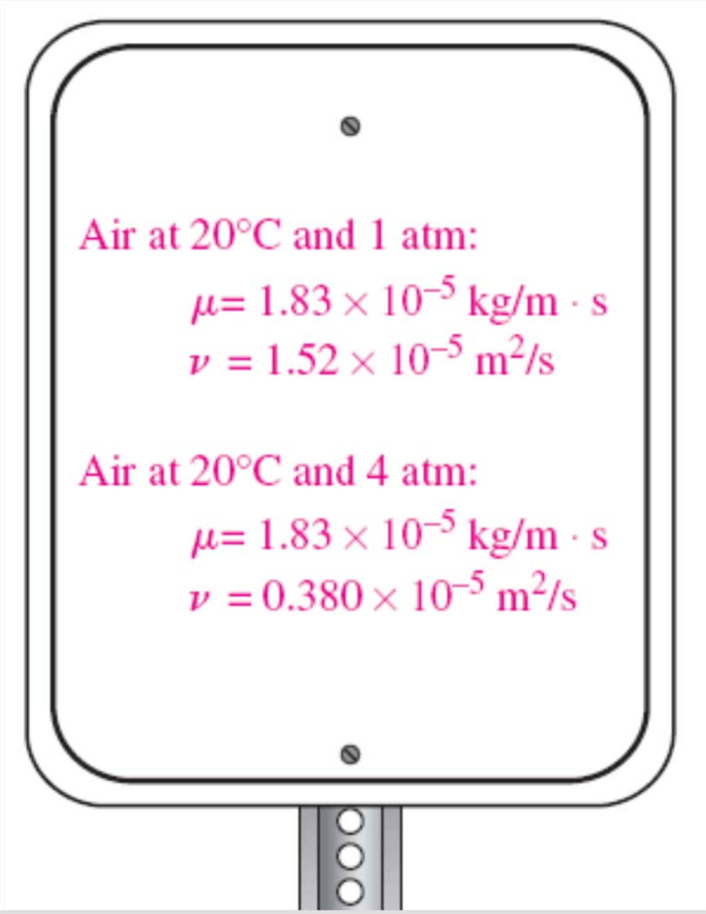
For *gases*, this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure.

$$\mu = \frac{aT^{1/2}}{1 + b/T}$$

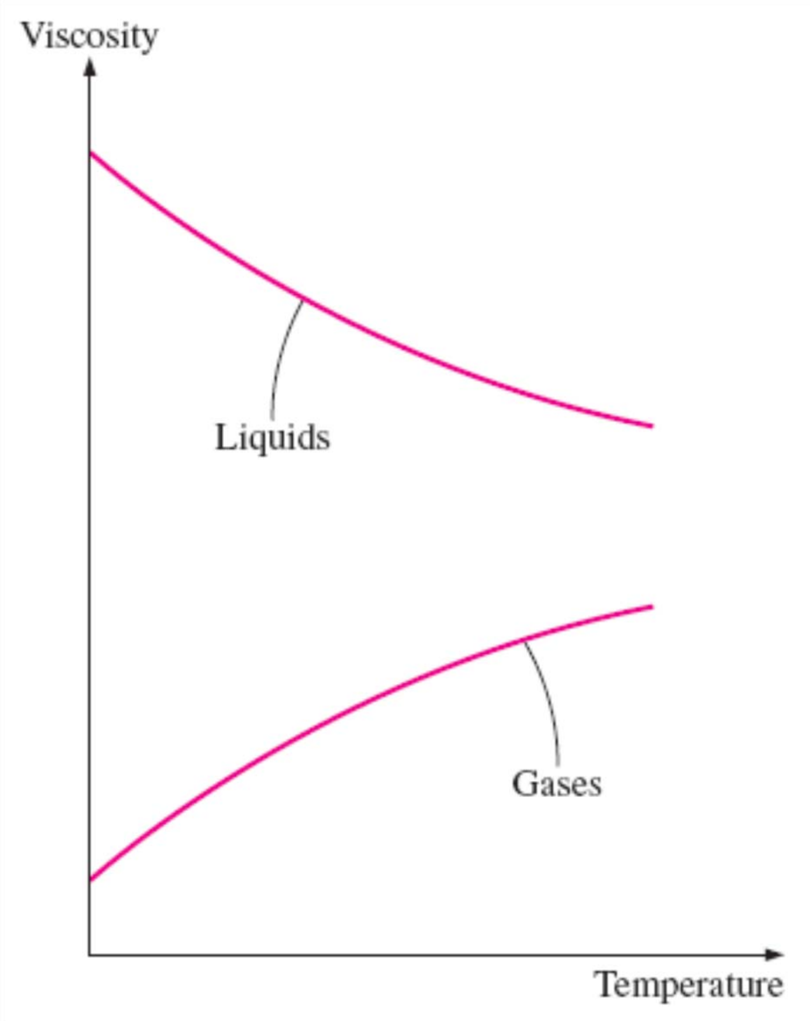
For gases:

$$\mu = a10^{b/(T-c)}$$

For liquids



Dynamic viscosity, in general, does not depend on pressure, but kinematic viscosity does.



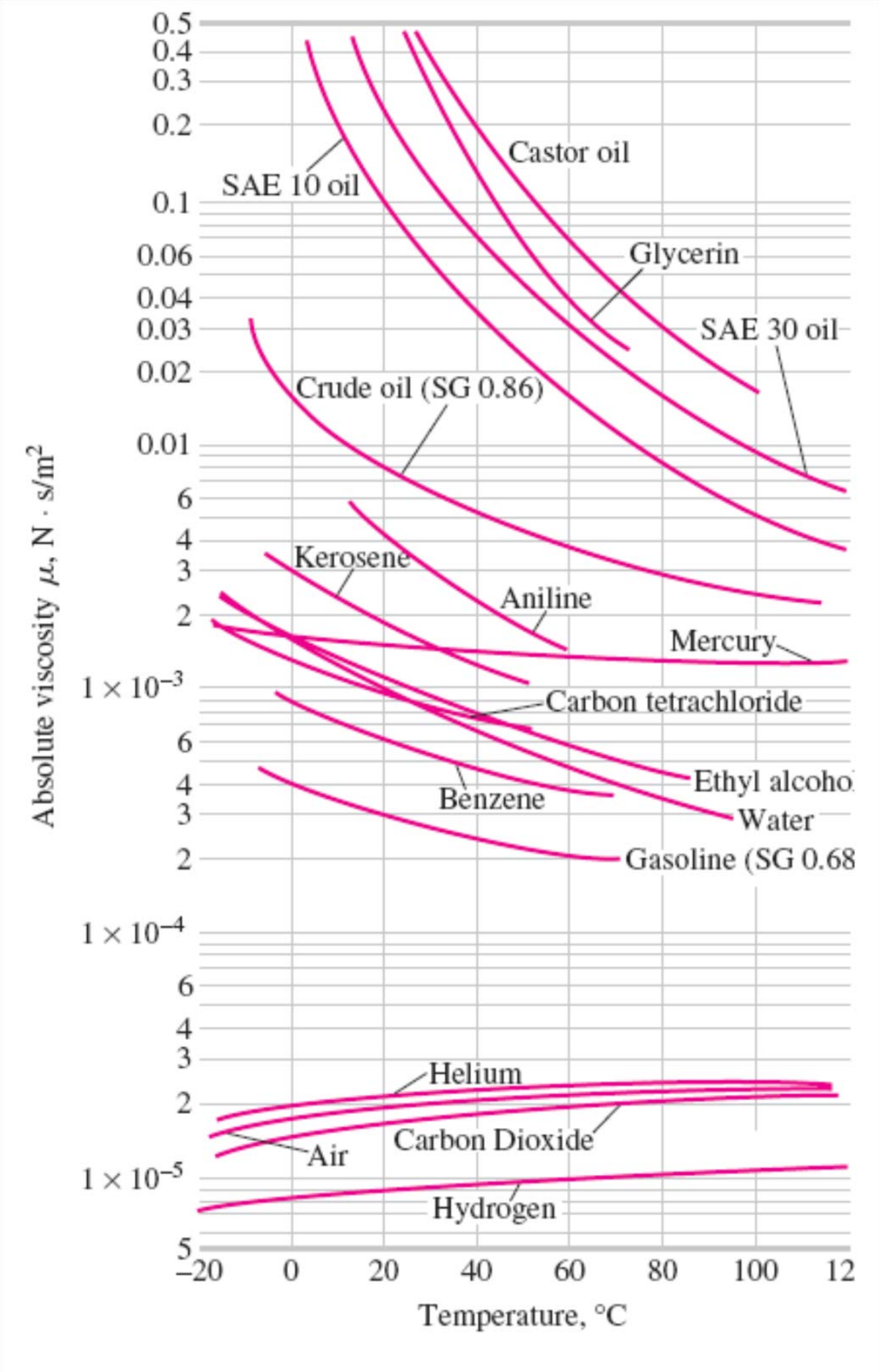
The viscosity of liquids decreases and the viscosity of gases increases with temperature.

The viscosity of a fluid is directly related to the pumping power needed to transport a fluid in a pipe or to move a body through a fluid.

Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature.

In a liquid, the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely.

In a gas, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time and therefore in greater resistance to flow.

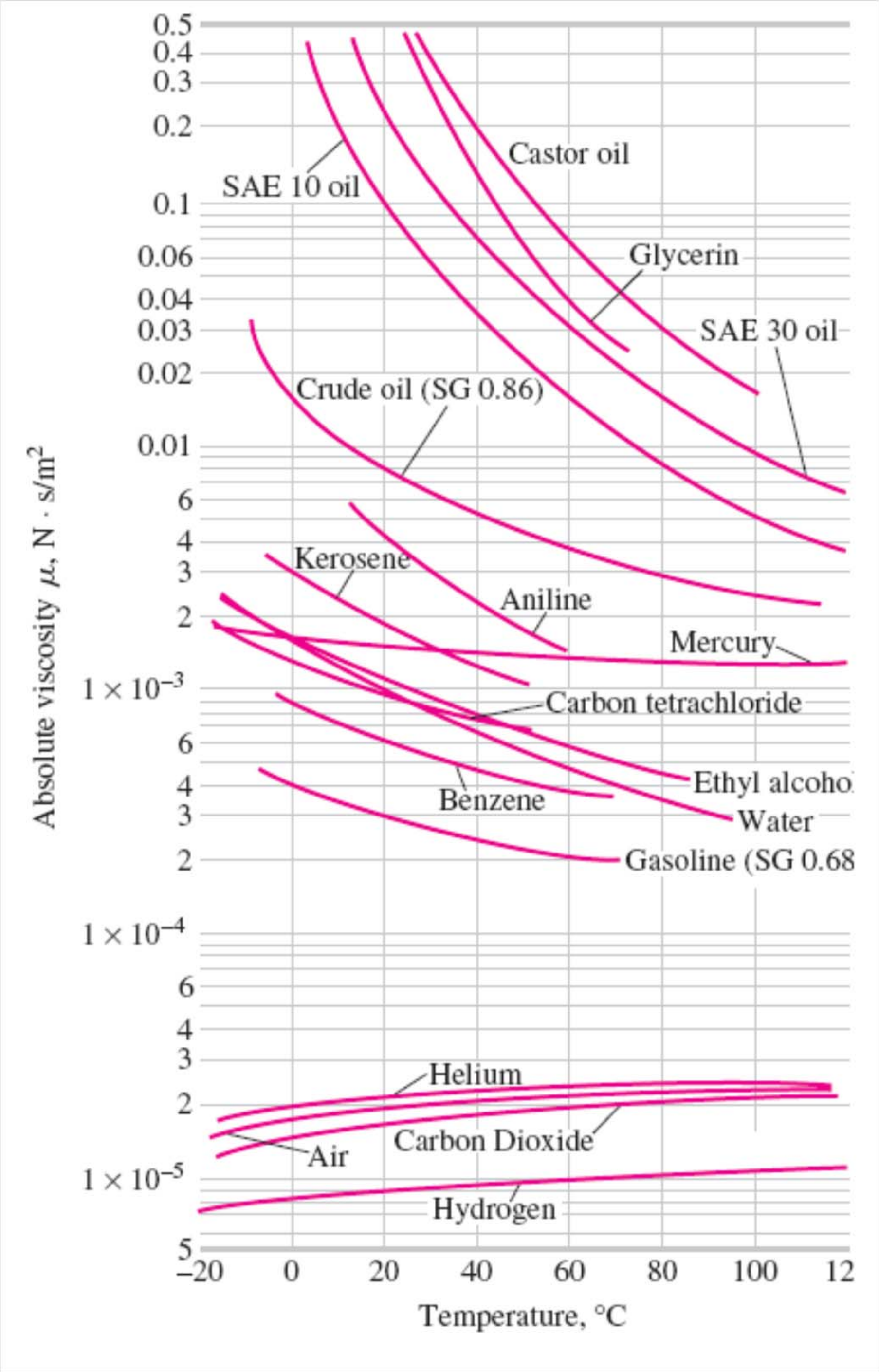


The variation of dynamic (absolute) viscosity of common fluids with temperature at 1 atm

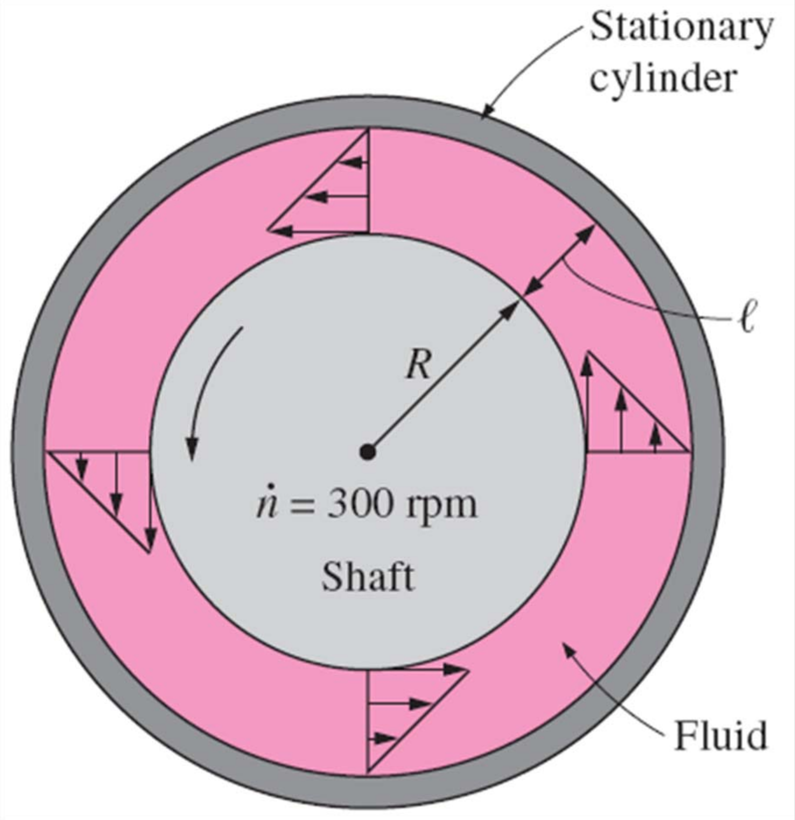
(1 N·s/m^2 = 1kg/m·s = 0.020886 lbf·s/ft^2)

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic Viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088



The variation of dynamic (absolute) viscosity of common fluids with temperature at 1 atm (1 N·s/m^2 = 1 kg/m·s = 0.020886 lbf·s/ft^2).



L length of the cylinder

$\dot{n}$ number of revolutions per unit time

$$\mathrm{T} = FR = \mu \frac{2\pi R^3 \omega L}{\ell} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell}$$

This equation can be used to calculate the viscosity of a fluid by measuring torque at a specified angular velocity.

Therefore, two concentric cylinders can be used as a ***viscometer***, a device that measures viscosity.

EXAMPLE 2–5 Determining the Viscosity of a Fluid

The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders (Fig. 2–27). The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N · m. Determine the viscosity of the fluid.

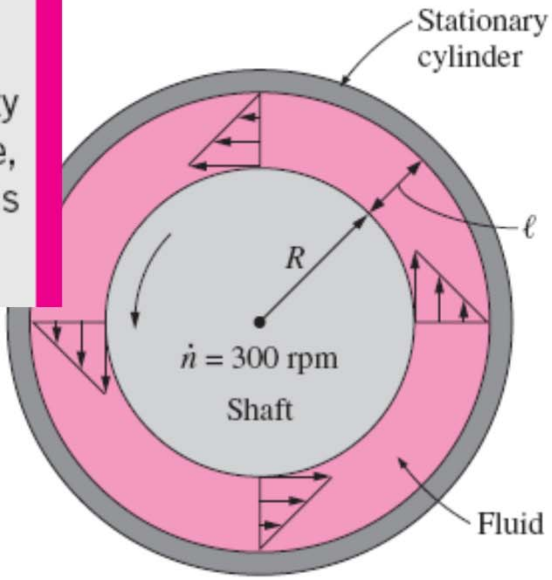
SOLUTION The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in the fluid. **2** The viscous effects on the two ends of the inner cylinder are negligible.

Analysis The velocity profile is linear only when the curvature effects are negligible, and the profile can be approximated as being linear in this case since $\ell/R = 0.025 \ll 1$. Solving Eq. 2–38 for viscosity and substituting the given values, the viscosity of the fluid is determined to be

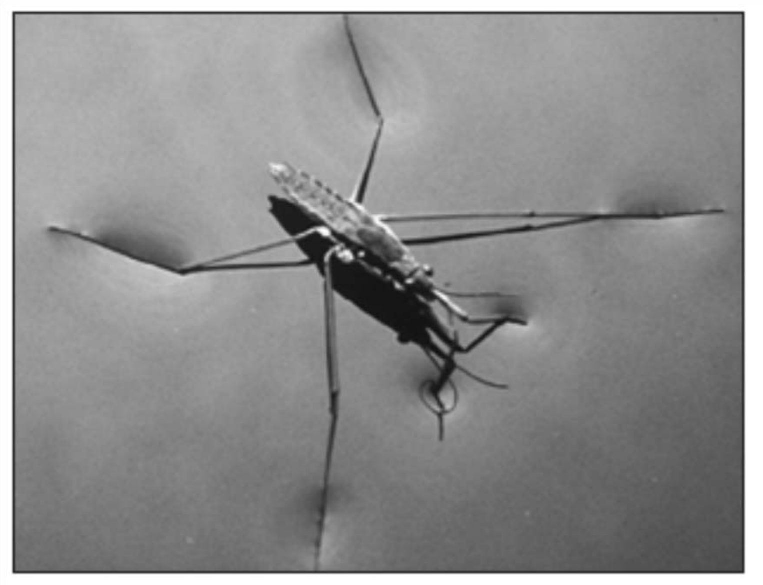
$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.8\ \mathrm{N \cdot m})(0.0015\ \mathrm{m})}{4\pi^2 (0.06\ \mathrm{m})^3 (300/60\ 1/\mathrm{s})(0.4\ \mathrm{m})} = \mathbf{0.158\ N \cdot s/m^2}$$

Discussion Viscosity is a strong function of temperature, and a viscosity value without a corresponding temperature is of little usefulness. Therefore, the temperature of the fluid should have also been measured during this experiment, and reported with this calculation.

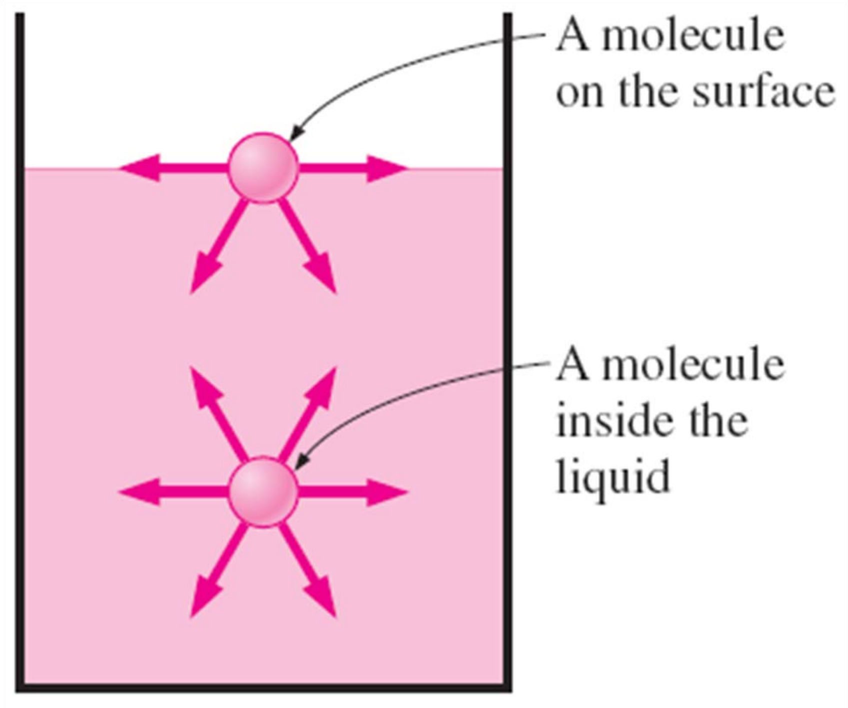


2–7 ■ SURFACE TENSION AND CAPILLARY EFFECT

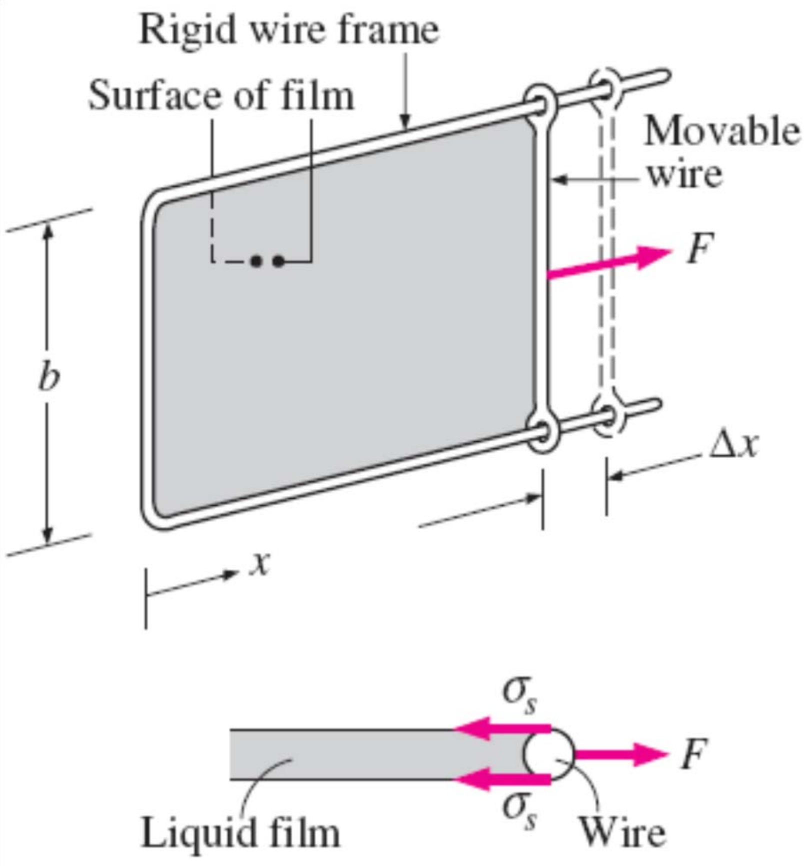
- Liquid droplets behave like small balloons filled with the liquid on a solid surface, and the surface of the liquid acts like a stretched elastic membrane under tension.
- The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid.
- The magnitude of this force per unit length is called **surface tension** (or *coefficient of surface tension*) and is usually expressed in the unit N/m.
- This effect is also called *surface energy* [per unit area] and is expressed in the equivalent unit of N · m/m^2.



Some consequences of surface tension.



Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.



Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length *b*.

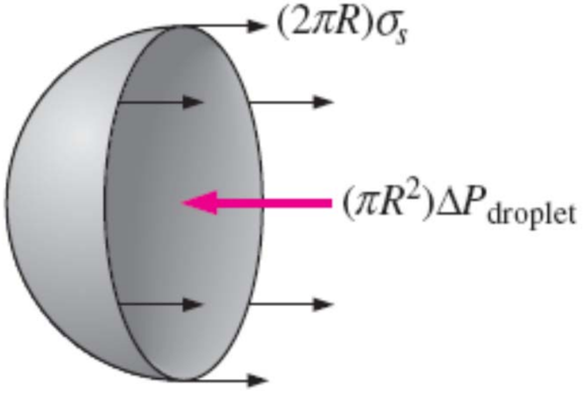
$$\sigma_s = \frac{F}{2b}$$

$$W = \text{Force} \times \text{Distance} = F\,\Delta x = 2b\sigma_s\,\Delta x = \sigma_s\,\Delta A$$

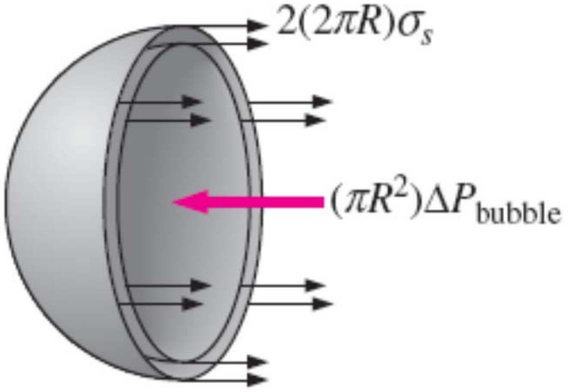
Surface tension: The work done per unit increase in the surface area of the liquid.

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tension σ_s, N/m*
†Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028



(*a*) Half a droplet or air bubble



(*b*) Half a soap bubble

The free-body diagram of half a droplet or air bubble and half a soap bubble.

Droplet or air bubble:

$$(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \rightarrow \Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R}$$

Soap bubble:

$$2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \rightarrow \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R}$$

Capillary effect: The rise or fall of a liquid in a small-diameter tube inserted into the liquid.

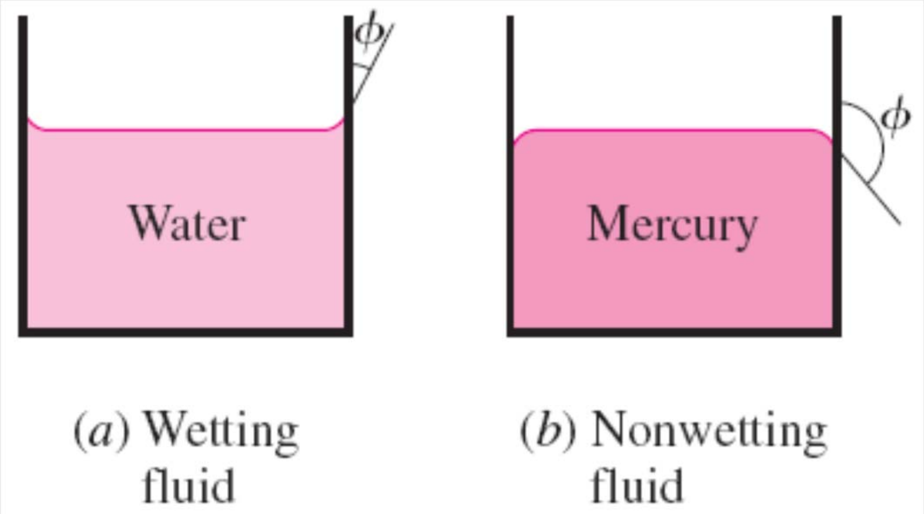
Capillaries: Such narrow tubes or confined flow channels.

The capillary effect is partially responsible for the rise of water to the top of tall trees.

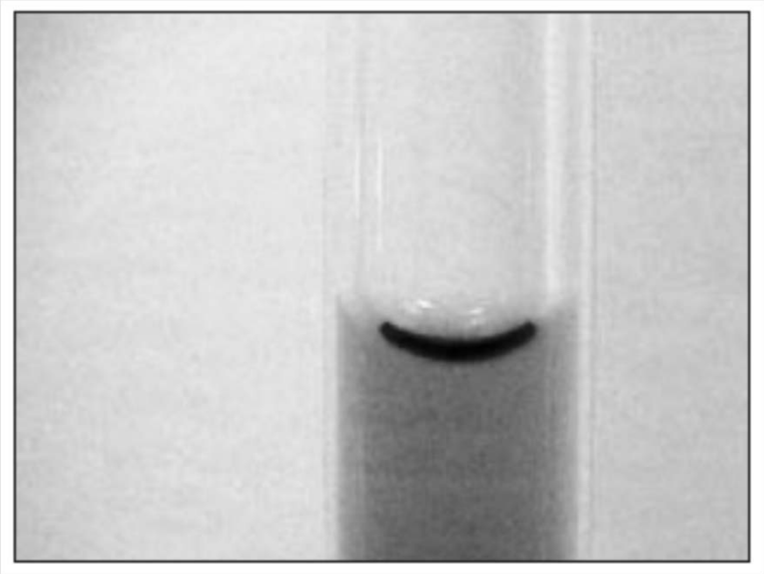
Meniscus: The curved free surface of a liquid in a capillary tube.

Capillary Effect

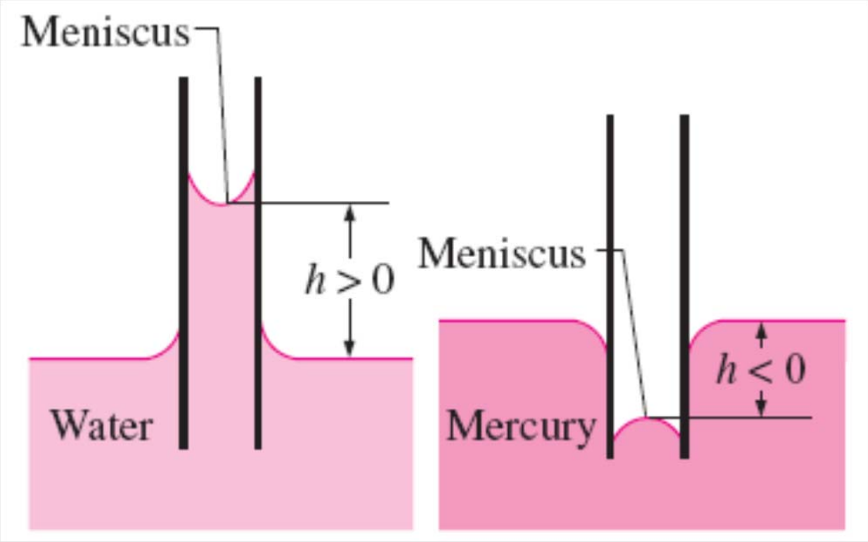
The strength of the capillary effect is quantified by the **contact** (or *wetting*) **angle**, defined as *the angle that the tangent to the liquid surface makes with the solid surface at the point of contact*.



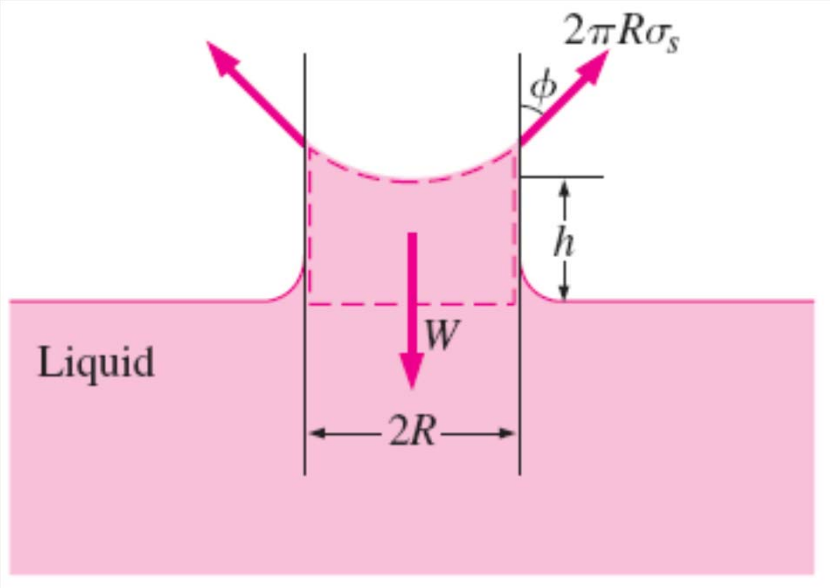
The contact angle for wetting and nonwetting fluids.



The meniscus of colored water in a 4-mm-inner-diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle.



The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.



The forces acting on a liquid column that has risen in a tube due to the capillary effect.

Capillary rise: $$h = \frac{2\sigma_s}{\rho g R}\cos\phi \qquad (R = \text{constant})$$

- Capillary rise is inversely proportional to the radius of the tube and density of the liquid.

EXAMPLE 2–6 The Capillary Rise of Water in a Tube

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube (Fig. 2–36).

SOLUTION The rise of water in a slender tube as a result of the capillary effect is to be determined.

Assumptions **1** There are no impurities in the water and no contamination on the surfaces of the glass tube. **2** The experiment is conducted in atmospheric air.

Properties The surface tension of water at 20°C is 0.073 N/m (Table 2–4). The contact angle of water with glass is approximately 0° (from preceding text). We take the density of liquid water to be 1000 kg/m^3.

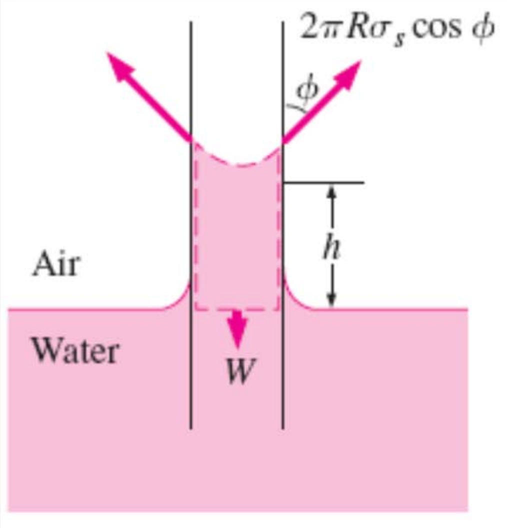
Analysis The capillary rise is determined directly from Eq. 2–42 by substituting the given values, yielding

$$h = \frac{2\sigma_s}{\rho g R}\cos\phi = \frac{2(0.073\ \text{N/m})}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3 \times 10^{-3}\text{m})}(\cos 0°)\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)$$

$$= 0.050\ \text{m} = \mathbf{5.0\ cm}$$

Therefore, water rises in the tube 5 cm above the liquid level in the cup.

Discussion Note that if the tube diameter were 1 cm, the capillary rise would be 0.3 mm, which is hardly noticeable to the eye. Actually, the capillary rise in a large-diameter tube occurs only at the rim. The center does not rise at all. Therefore, the capillary effect can be ignored for large-diameter tubes.



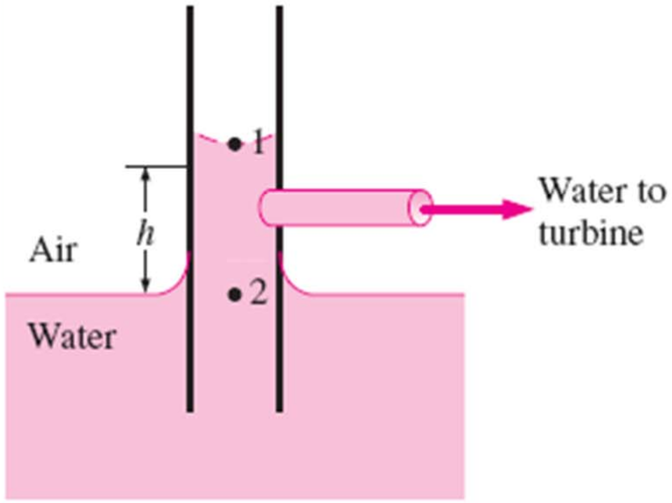
EXAMPLE 2–7 Using Capillary Rise to Generate Power in a Hydraulic Turbine

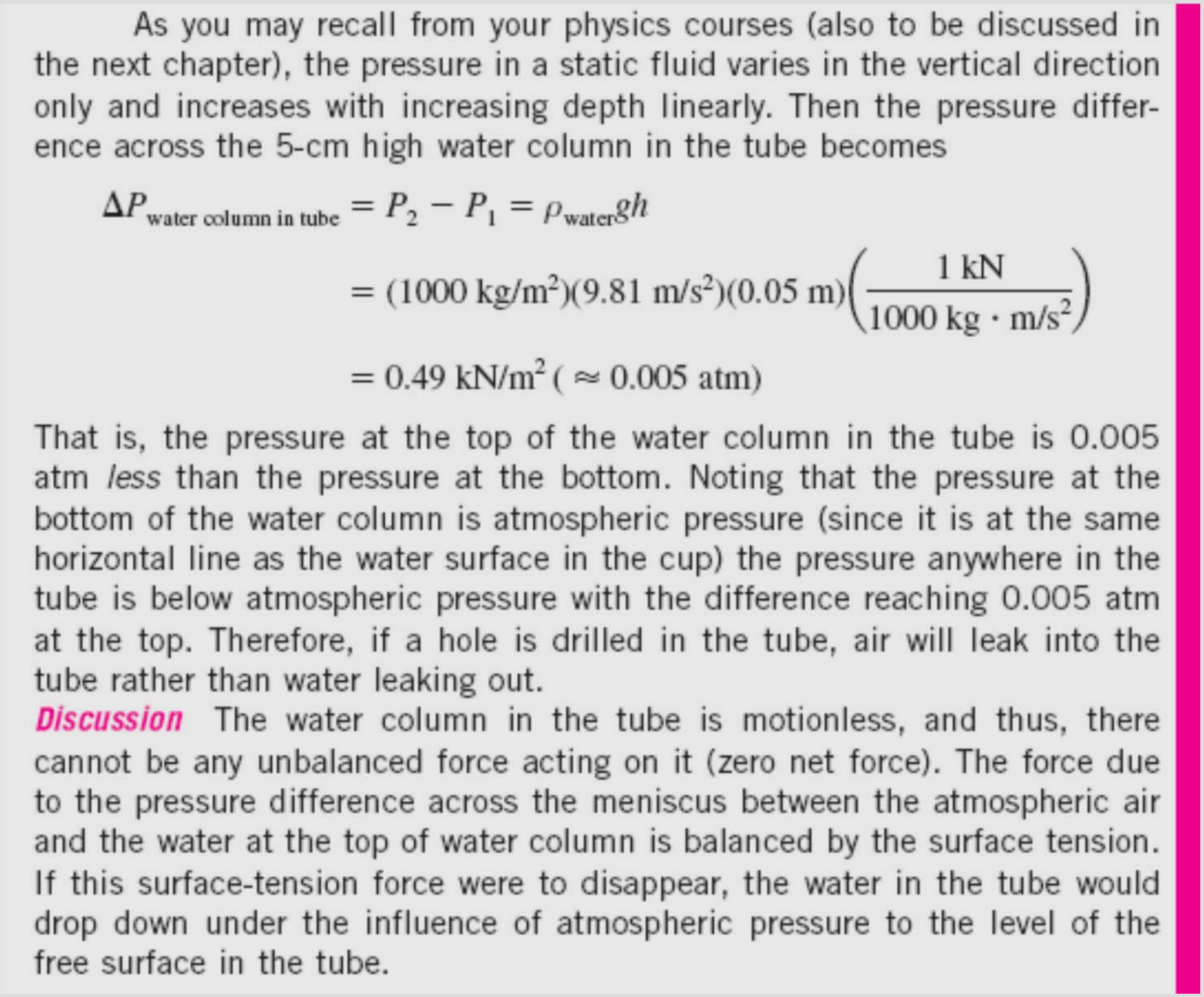
Reconsider Example 2-6. Realizing that water rises by 5 cm under the influence of surface tension without requiring any energy input from an external source, a person conceives the idea that power can be generated by drilling a hole in the tube just below the water level and feeding the water spilling out of the tube into a turbine (Fig. 2-37). The person takes this idea even further by suggesting that a series of tube banks can be used for this purpose and cascading can be incorporated to achieve practically feasible flow rates and elevation differences. Determine if this idea has any merit.

SOLUTION Water that rises in tubes under the influence of the capillary effect is to be used to generate power by feeding it into a turbine. The validity of this suggestion is to be evaluated.

Analysis The proposed system may appear like a stroke of genius, since the commonly used hydroelectric power plants generate electric power by simply capturing the potential energy of elevated water, and the capillary rise provides the mechanism to raise the water to any desired height without requiring any energy input.

When viewed from a thermodynamic point of view, the proposed system immediately can be labeled as a perpetual motion machine (PMM) since it continuously generates electric power without requiring any energy input. That is, the proposed system creates energy, which is a clear violation of the first law of thermodynamics or the conservation of energy principle, and it does not warrant any further consideration. But the fundamental principle of conservation of energy did not stop many from dreaming about being the first to prove nature wrong, and to come up with a trick to permanently solve the world's energy problems. Therefore, the impossibility of the proposed system should be demonstrated.





As you may recall from your physics courses (also to be discussed in the next chapter), the pressure in a static fluid varies in the vertical direction only and increases with increasing depth linearly. Then the pressure difference across the 5-cm high water column in the tube becomes

$$\begin{aligned}\Delta P_{\text{water column in tube}} &= P_2 - P_1 = \rho_{\text{water}} g h \\ &= (1000\ \text{kg/m}^2)(9.81\ \text{m/s}^2)(0.05\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) \\ &= 0.49\ \text{kN/m}^2\ (\approx 0.005\ \text{atm})\end{aligned}$$

That is, the pressure at the top of the water column in the tube is 0.005 atm *less* than the pressure at the bottom. Noting that the pressure at the bottom of the water column is atmospheric pressure (since it is at the same horizontal line as the water surface in the cup) the pressure anywhere in the tube is below atmospheric pressure with the difference reaching 0.005 atm at the top. Therefore, if a hole is drilled in the tube, air will leak into the tube rather than water leaking out.

Discussion The water column in the tube is motionless, and thus, there cannot be any unbalanced force acting on it (zero net force). The force due to the pressure difference across the meniscus between the atmospheric air and the water at the top of water column is balanced by the surface tension. If this surface-tension force were to disappear, the water in the tube would drop down under the influence of atmospheric pressure to the level of the free surface in the tube.

Summary

- Introduction
  - ✓ Continuum
- Density and Specific Gravity
  - ✓ Density of Ideal Gases
- Vapor Pressure and Cavitation
- Energy and Specific Heats
- Compressibility and Speed of Sound
  - ✓ Coefficient of Compressibility
  - ✓ Coefficient of Volume Expansion
  - ✓ Speed of Sound and Mach Number
- Viscosity
- Surface Tension and Capillary Effect